Efficient Power Minimization for MIMO Broadcast Channels with BD-GMD

Winston Ho and Ying-Chang Liang

Institute for Infocomm Research
(I2R)
Singapore

Overview

- Optimal Power Minimization
- Preliminaries: GMD and BD-GMD
- ZF-based Power Minimization
- Suboptimal Methods
- Simulations
- Conclusion

Optimal Power Minimization

- Objective:
  - To minimize the transmit power for the MIMO broadcast channel,
  - given user rate requirements,
  - using Dirty Paper Coding.
- Application:
  - Mimimize interference to neighbour cells.
  - Users at varying distances.
- Cases:
  - Interference-Balancing (IB).
  - Zero-Forcing (ZF).

Optimal Power Minimization

- Interference-Balancing Case
  - IUI, noise
  - Better performance than ZF in low SNR region
  - Higher complexity than ZF case
  - Many iterations
  - Each iter. ↑ computations
  - No. iter. random

Optimal Power Minimization

- Zero-Forcing Case
  - Lower complexity than the IB case
- User ordering affects transmission power
  - Search over K! encoding orders
    - Limited predictable complexity
  - Suboptimal methods with much reduced complexity
    - close to ZF-optimal power

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Preliminaries

Transmission Strategies for Single-User MIMO

- Singular Value Decomposition (SVD) $\mathbf{H = USV^H}$
  - ➢ Different constellations for each subchannel
- Geometric Mean Decomposition (**GMD**)[1] $\mathbf{H = QRP^H}$
  - ➢ R is triangular, equal diagonal
  - ➢ Same constellation for every subchannel
  - ➢ Low complexity
  - ➢ Good BER

[1] Y. Jiang, J. Li and W. W. Hager, "Joint Transceiver Design for MIMO Communications Using Geometric Mean Decomposition,"
IEEE Trans. Signal Processing, vol. 53, no. 10, pp. 3791-3803, Oct. 2005.

Preliminaries

Block-Diagonal GMD for Multi-User MIMO

$$H = P\,L\,Q^H$$

- P: Block Diagonal & Unitary
- L: Lower Triangular
- Q^H: Unitary

$$\begin{bmatrix} \mathbf{P}_1 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{P}_K \end{bmatrix}$$

Each P_i is unitary.

$$\begin{bmatrix} \mathbf{L}_1 & 0 & \dots & 0 \\ X & \mathbf{L}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X & X & \dots & \mathbf{L}_K \end{bmatrix}$$

Each L_i is equal diagonal.

Block-equal-diagonal

S. Lin, W. W. L. Ho, and Y.-C. Liang, "Block-diagonal Geometric Mean Decomposition (BD-GMD) for Multiuser MIMO Broadcast Channels," *Int. Symp. Personal, Indoor and Mobile Radio Commun.*, Helsinki, 11–14 Sep. 2006.

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Power Minimization Given Fixed Encoding Order

- Rate requirements ⇆ SNR requirements
  - rate = $\log_2(1+SNR)$
- # transmit antennas = N_T
- # receive antennas = $n_1, n_2, \ldots, n_K$

  sum=$N_R < N_T$
- Multiplexing: user i has n_i data subchannels
- SNR req'm for each subchannel = γ_i

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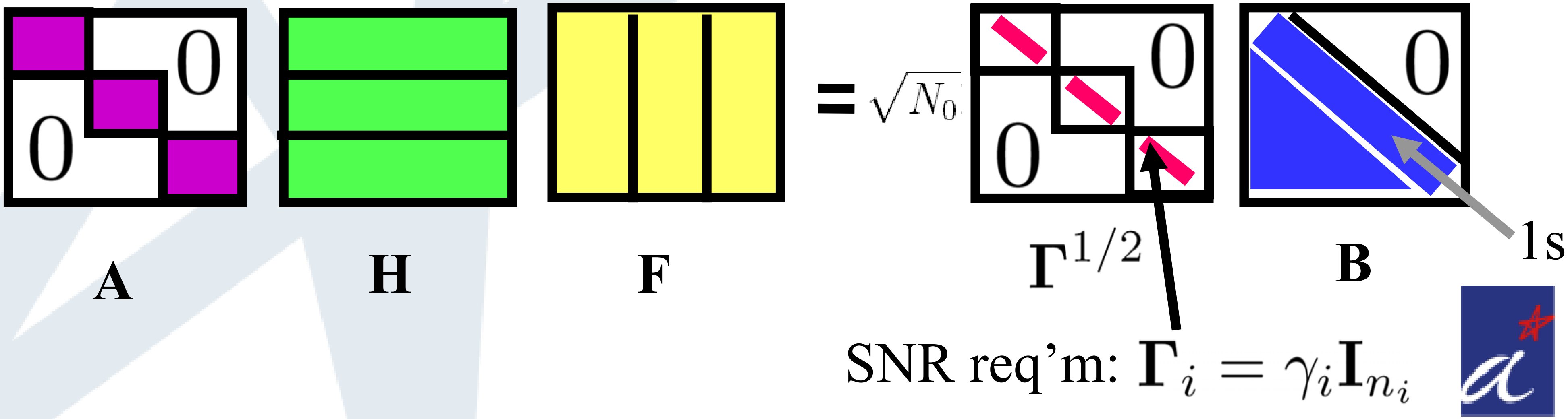
Power Minimization Given Fixed Encoding Order

- Optimization problem:

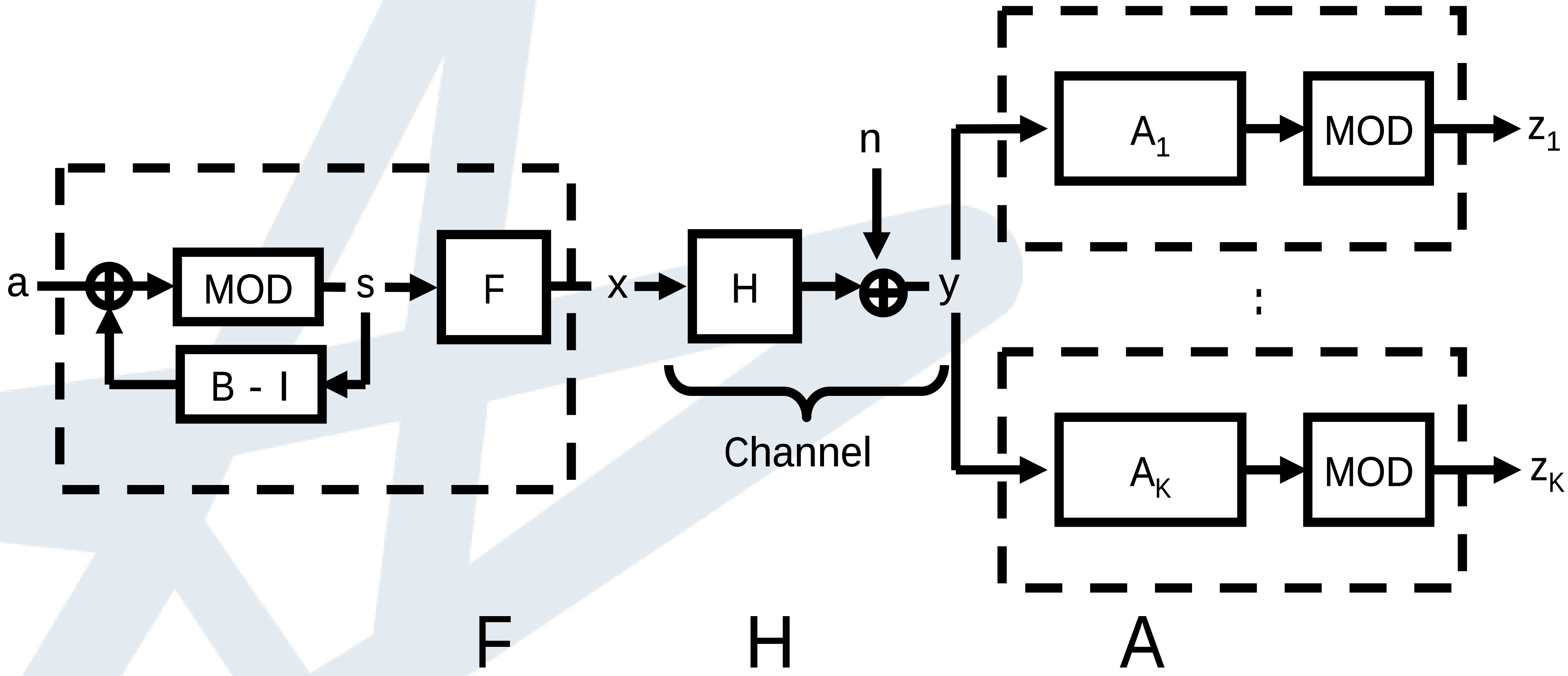
$$\begin{aligned} &\texttt{minimize} && \mathrm{Tr}(\mathbf{F}^H\mathbf{F}) \\ &\texttt{subject to} && \mathbf{AHF} = \sqrt{N_0}\mathbf{\Gamma}^{1/2}\mathbf{B} \\ & && \mathbf{B} \in \mathbb{L}\ ,\ \mathbf{A} \in \mathbb{B} \\ & && \|\mathbf{A}(i,:)\| = 1 \qquad \text{for } 1 \le i \le N_R\ . \end{aligned}$$



Power Minimization Given Fixed Encoding Order

- Solution:
  - BD-GMD: $\mathbf{H} = \mathbf{P}\mathbf{L}\mathbf{Q}^H$
  - let $\mathbf{\Lambda} = \mathrm{diag}(\mathbf{L})$
  - $$\mathbf{\Omega} = \sqrt{N_0}\mathbf{\Gamma}^{1/2}\mathbf{\Lambda}^{-1}, \quad \mathbf{F} = \mathbf{Q}\mathbf{\Omega},$$ $$\mathbf{B} = \mathbf{\Omega}^{-1}\mathbf{\Lambda}^{-1}\mathbf{L}\mathbf{\Omega}, \quad \mathbf{A} = \mathbf{P}^H.$$
  - Minimum power = $E_s = \mathrm{Tr}(\mathbf{F}^H\mathbf{F}) = \mathrm{Tr}(\mathbf{\Omega}^2)$

Transceiver Design

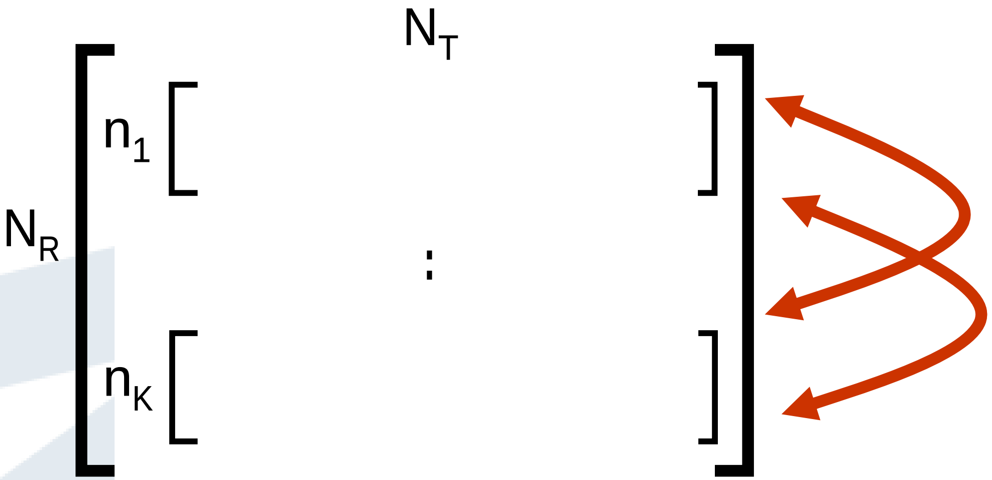


$$\mathbf{AHF} = \sqrt{N_0}\boldsymbol{\Gamma}^{1/2}\mathbf{B}$$

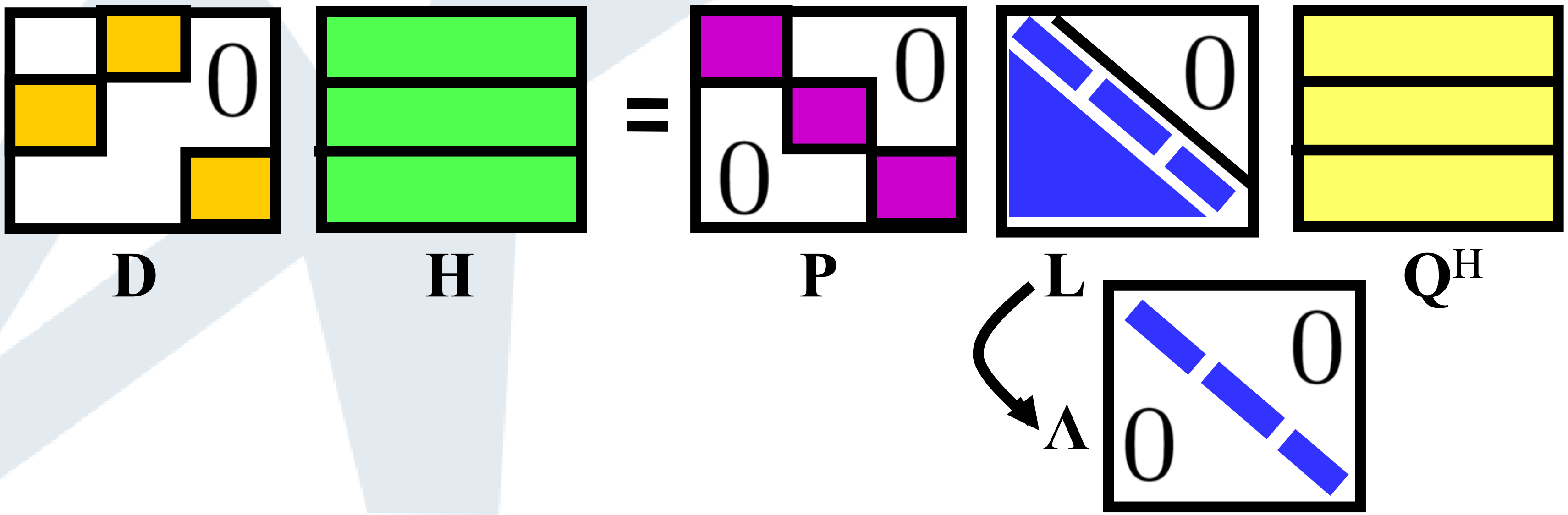
Optimal User Ordering

- Rearranging the Channel Matrix

$PLQ^H = H =$

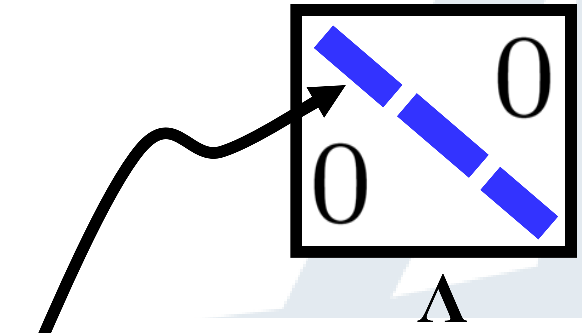


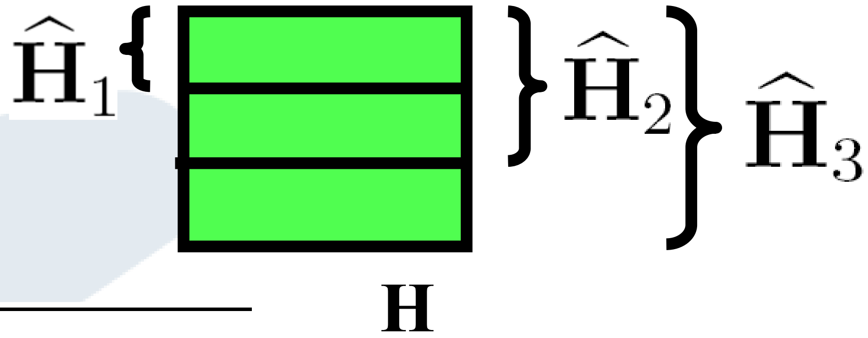
to achieve minimum transmit power.



Optimal User Ordering

- Transmit power $= \mathrm{Tr}(\boldsymbol{\Omega}^2) = N_0 \mathrm{Tr}\left(\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-2}\right)$



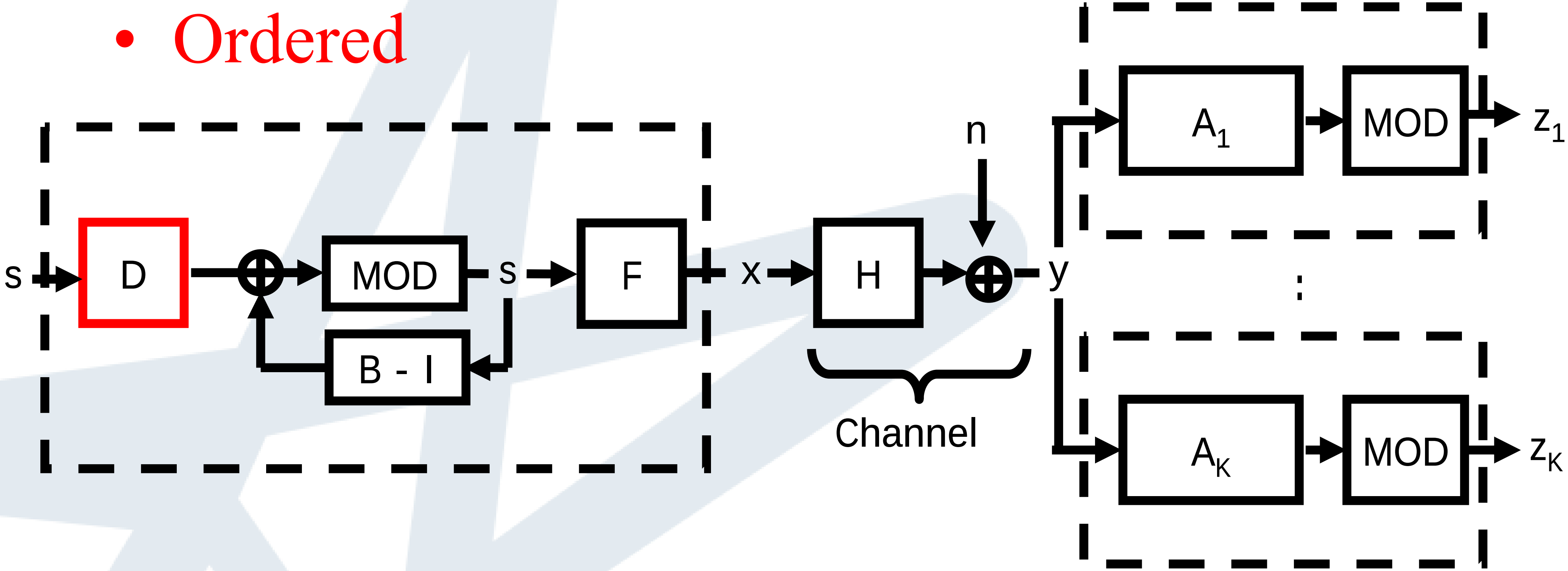


$$r_i = \sqrt[2n_i]{\det(\boldsymbol{\Lambda}_i^2)} = \sqrt[2n_i]{\frac{\det(\widehat{\mathbf{H}}_i \widehat{\mathbf{H}}_i^H)}{\det(\widehat{\mathbf{H}}_{i-1} \widehat{\mathbf{H}}_{i-1}^H)}}$$

- Optimal ordering – search all K! orderings
  - (K K!) determinant calculations

Transceiver Design

- Ordered



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Suboptimal User Ordering

- Principle
  - Successive selection of users
  - Top down manner
- Methods
  - 1. Successive Closest Match (SCM)
  - 2. Minimize r_i
  - 3. Minimize Channel Strength

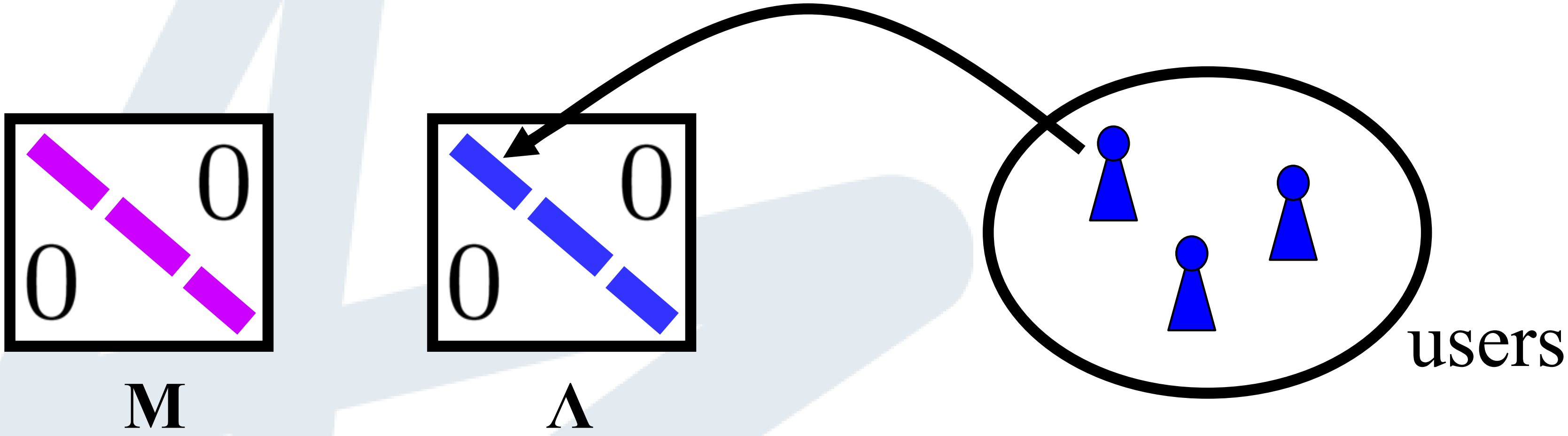
Method 1:
Successive Closest Match (SCM)

- det($\mathbf{\Omega}^2$) fixed
- To minimize Tr($\mathbf{\Omega}^2$) $\quad \mathrm{Tr}(\mathbf{\Omega}^2) = N_0 \mathrm{Tr}\left(\mathbf{\Gamma}\mathbf{\Lambda}^{-2}\right)$
- equal elements
- Best case $\mathbf{\Lambda} = k\,\mathbf{\Gamma}^{1/2}$ $\rightarrow$ $\mathbf{M}$ = desired matrix

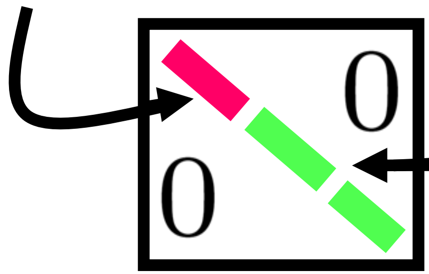
$$\mathbf{M} = \mathbf{\Gamma}^{1/2} \cdot \sqrt[2N_R]{\frac{\det(\mathbf{H}\mathbf{H}^H)}{\det(\mathbf{\Gamma})}}$$

- To minimize $\mathrm{Tr}\left(\mathbf{M}^2\mathbf{\Lambda}^{-2}\right)$

Method 1: Successive Closest Match (SCM)



- Choose $\mathbf{\Lambda}_i$ closest to $\mathbf{M}_i$
- Min $\mathrm{Tr}\left(\mathbf{M}^2\mathbf{\Lambda}^{-2}\right)$ while $\det\left(\mathbf{M}^2\mathbf{\Lambda}^{-2}\right) = 1$
- Min $n_1 \sqrt[n_1]{\det\left(\mathbf{M}_1^2\mathbf{\Lambda}_1^{-2}\right)} + \check{n}_2 \sqrt[\check{n}_2]{\det\left(\mathbf{\Lambda}_1^2\mathbf{M}_1^{-2}\right)}$



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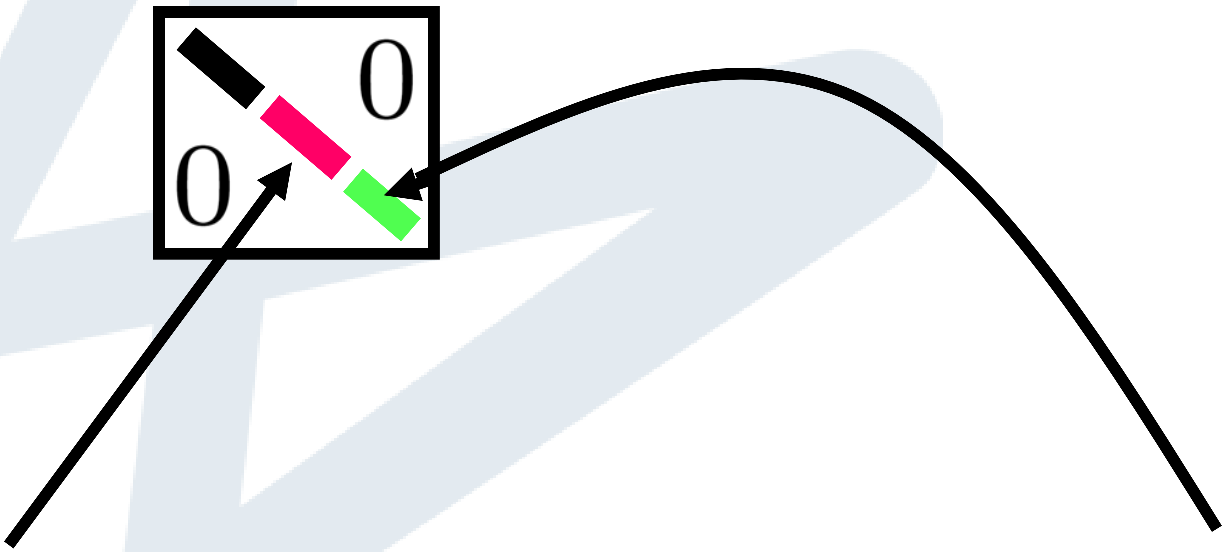
Method 1: Successive Closest Match (SCM)

- For user 2 onwards



- min

$$n_i \sqrt[n_i]{\det\left(\mathbf{M}_i^2 \mathbf{\Lambda}_i^{-2}\right)} + \check{n}_{i+1} \sqrt[\check{n}_{i+1}]{\det\left(\hat{\mathbf{\Lambda}}_{i-1}^2 \hat{\mathbf{M}}_{i-1}^{-2}\right) \det\left(\mathbf{\Lambda}_i^2 \mathbf{M}_i^{-2}\right)}$$

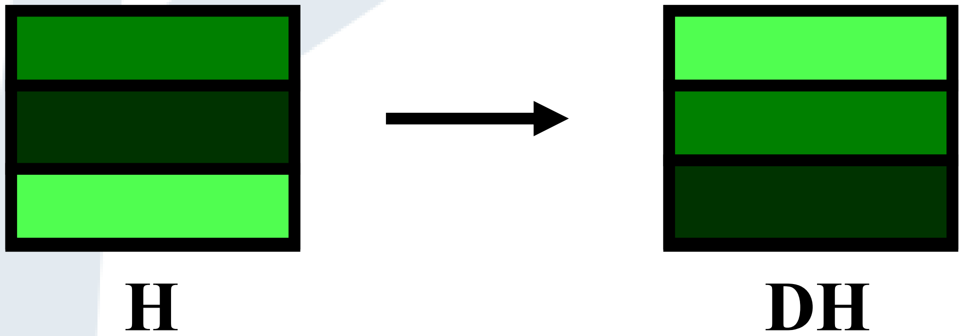
Method 2: Minimize r_i

- QR decomposition $\mathbf{P}^H\mathbf{H} = \mathbf{LQ}^H$
- r_i = diagonal elements of $\mathbf{L}$
- first element >> last element
- equal SNR req'm
- ↓ spread

Method 3:
Minimize Channel Strength

- Different channel strengths
- Weakest channel, encode first

- Minimize $\mathrm{Tr}(\mathbf{H}_i\mathbf{H}_i^H)/n_i$



Complexity

	No. of det. calc.
Optimal ZF power minimization	(K K!)
1. Successive Closest Match (SCM)	$\frac{K(K+1)}{2}$
2. Minimize r_i	$\frac{K(K+1)}{2}$
3. Minimize Channel Strength	0

BD-GMD applied to best order: K times GMD

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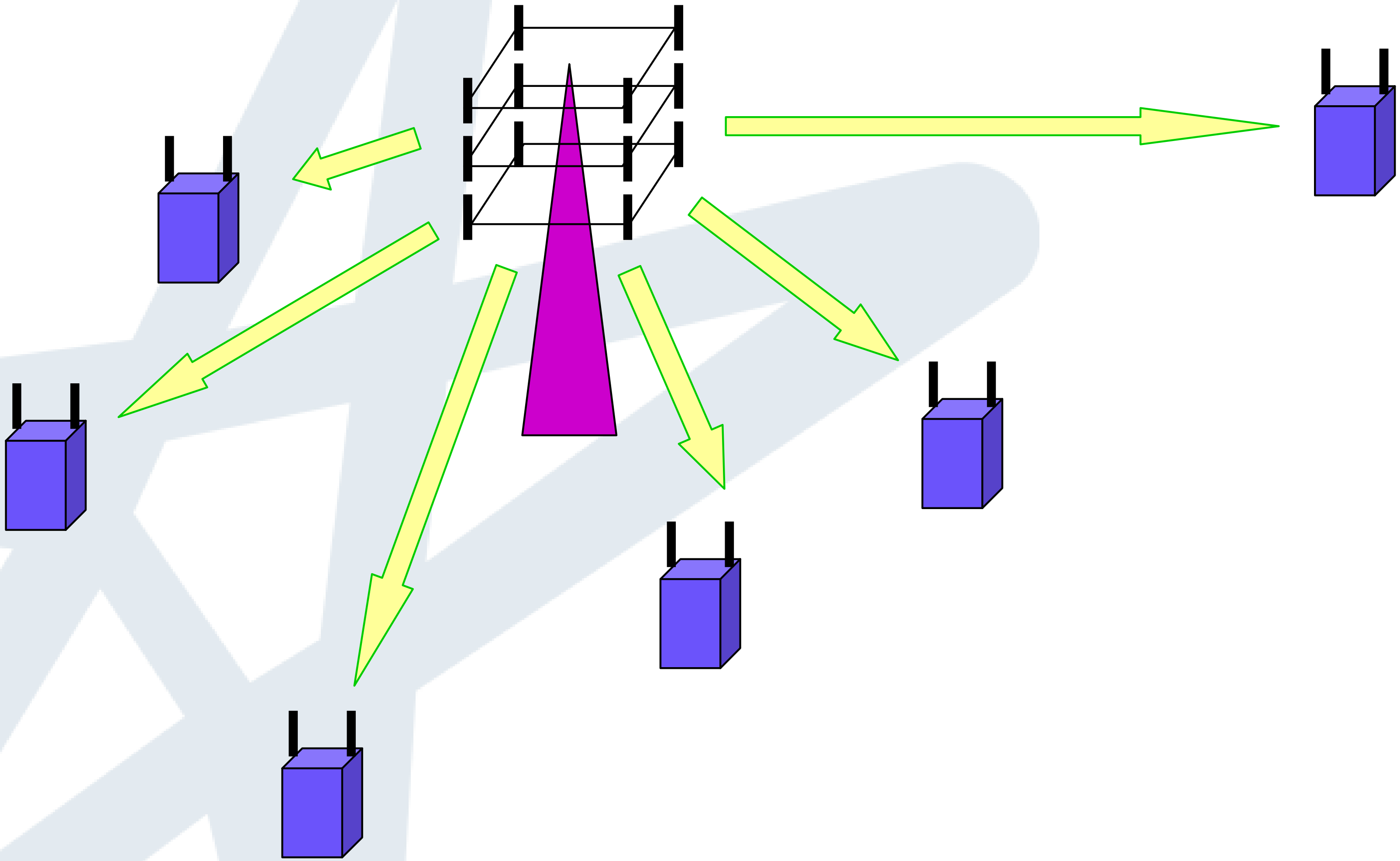
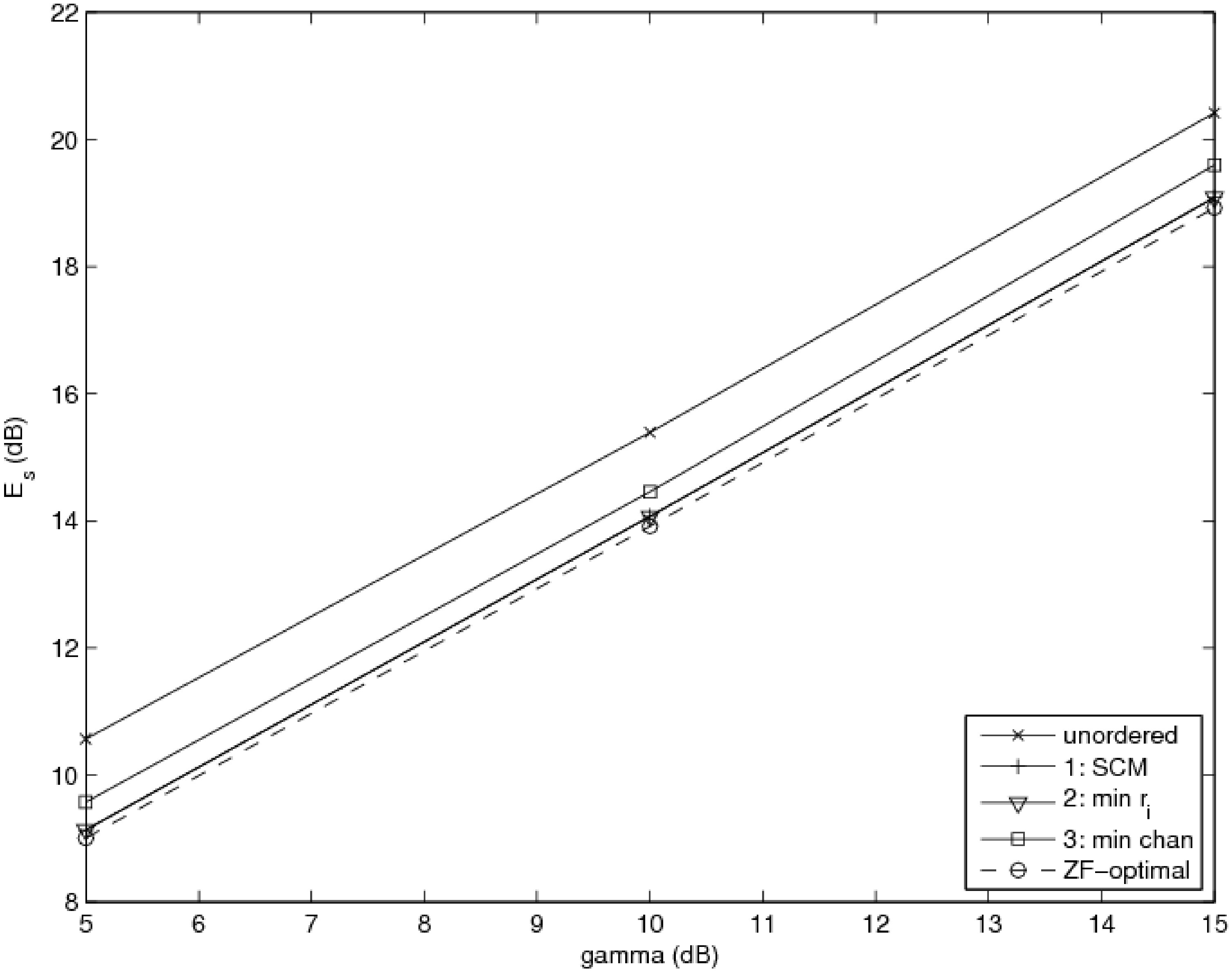
Simulation Results



Equal SNR requirements



$$12 \times [2, 2, 2, 2, 2, 2], \boldsymbol{\gamma} = [\gamma, \gamma, \gamma, \gamma, \gamma, \gamma]$$

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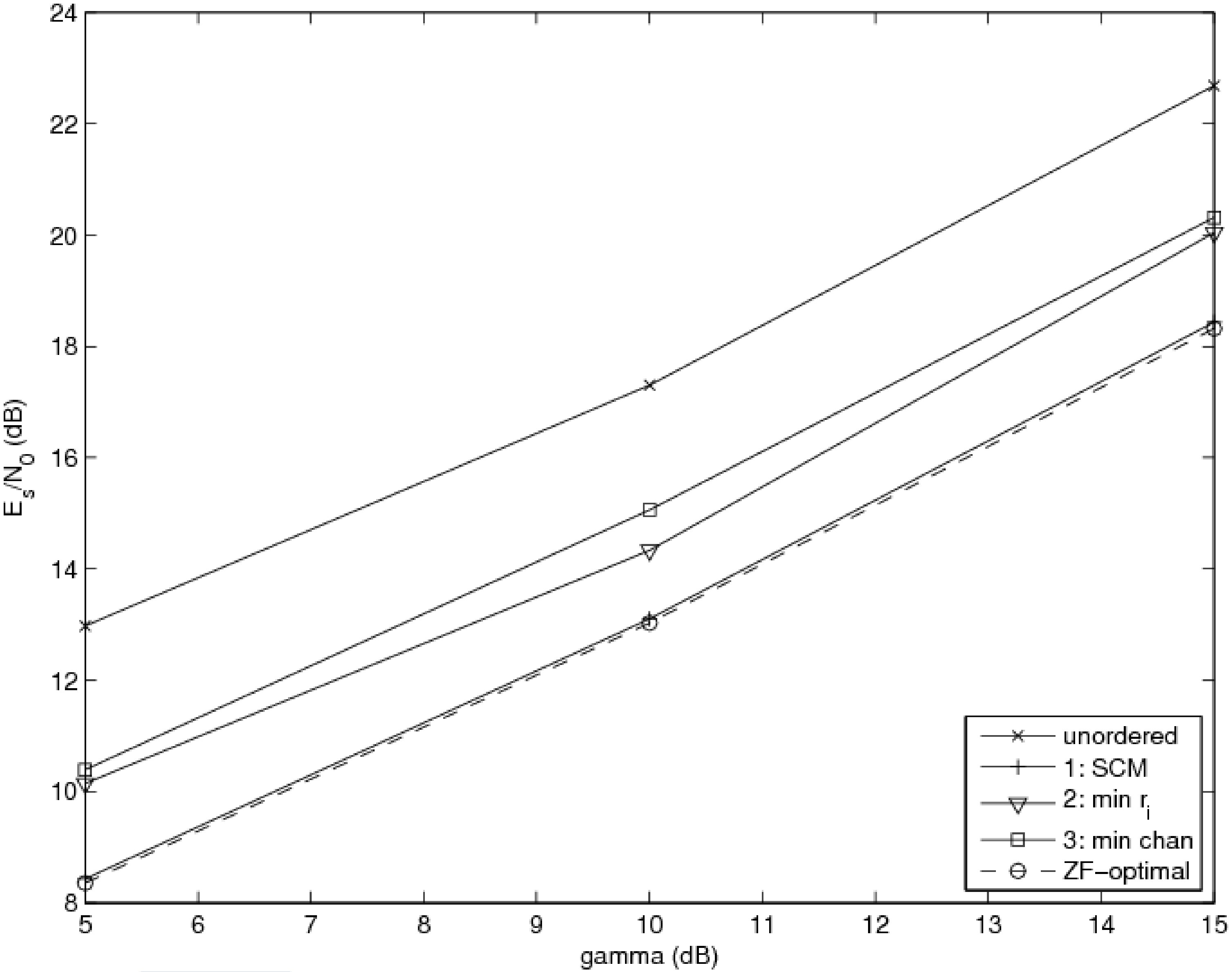
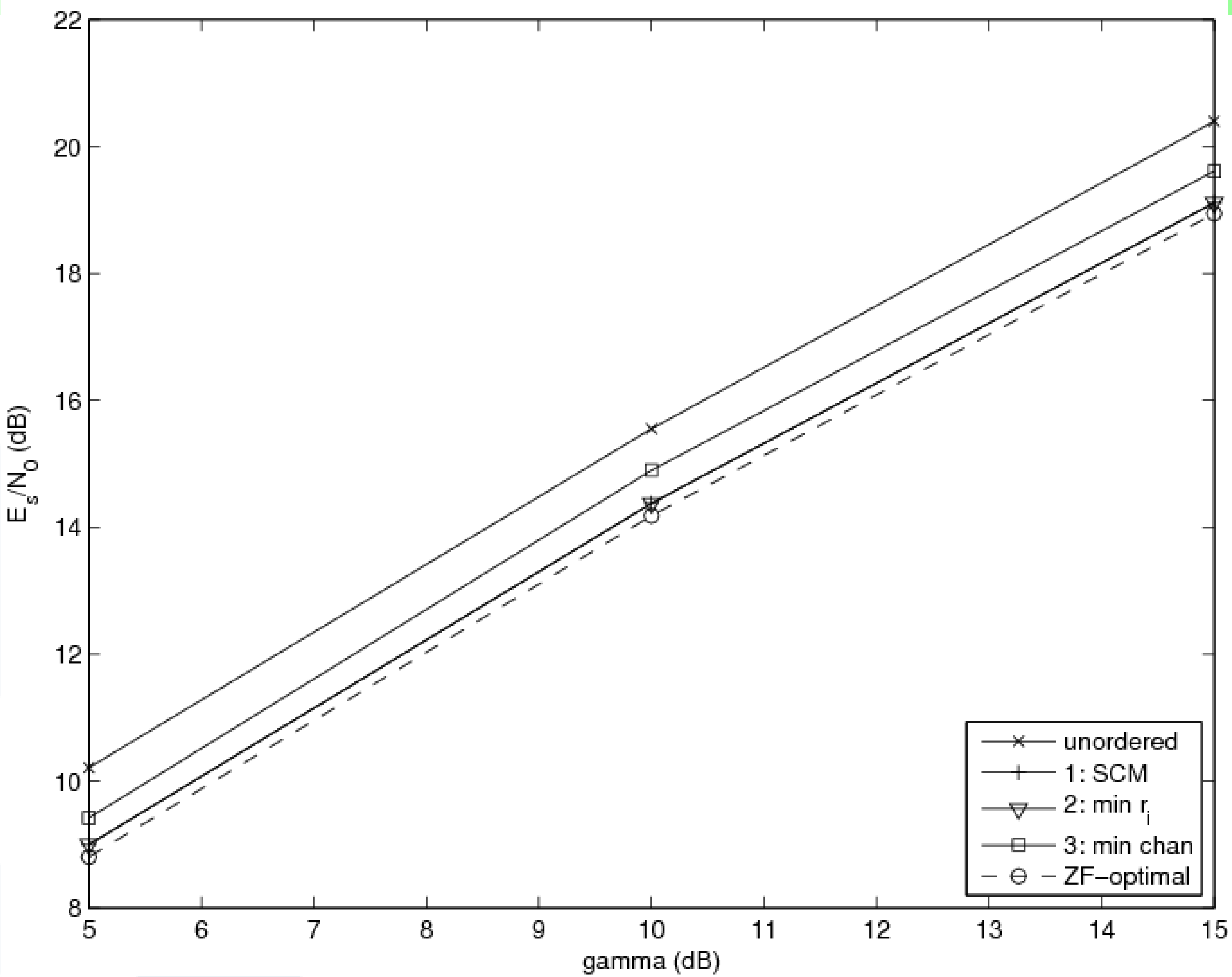
Unequal SNR requirements



$$12 \times [2,2,2,2,2,2],\ \boldsymbol{\gamma} = [\gamma/2, \gamma/2, \gamma, \gamma, 2\gamma, 2\gamma]$$

Unequal channel strengths



$$\mathbf{c} = [1.5, 1.5, 1, 1, 0.5, 0.5], 12 \times [2, 2, 2, 2, 2, 2],$$

$$\boldsymbol{\gamma} = [\gamma, \gamma, \gamma, \gamma, \gamma, \gamma]$$

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Conclusion

- ZF-based Power Minimization for MIMO Broadcast Channels
- Block-diagonal Geometric Mean Decomposition (BD-GMD)
- Equal SNR for all subchannels of each user
- Optimal ordering, non-iterative
- Suboptimal orderings