Block-Diagonal Geometric Mean Decomposition (BD-GMD) for Multiuser MIMO Broadcast

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Overview

- Single-user MIMO
- Multi-User MIMO
- BD-GMD
- Transceiver Design
- Considerations
  - User Ordering
  - Power Loading
- Simulations
- Conclusion

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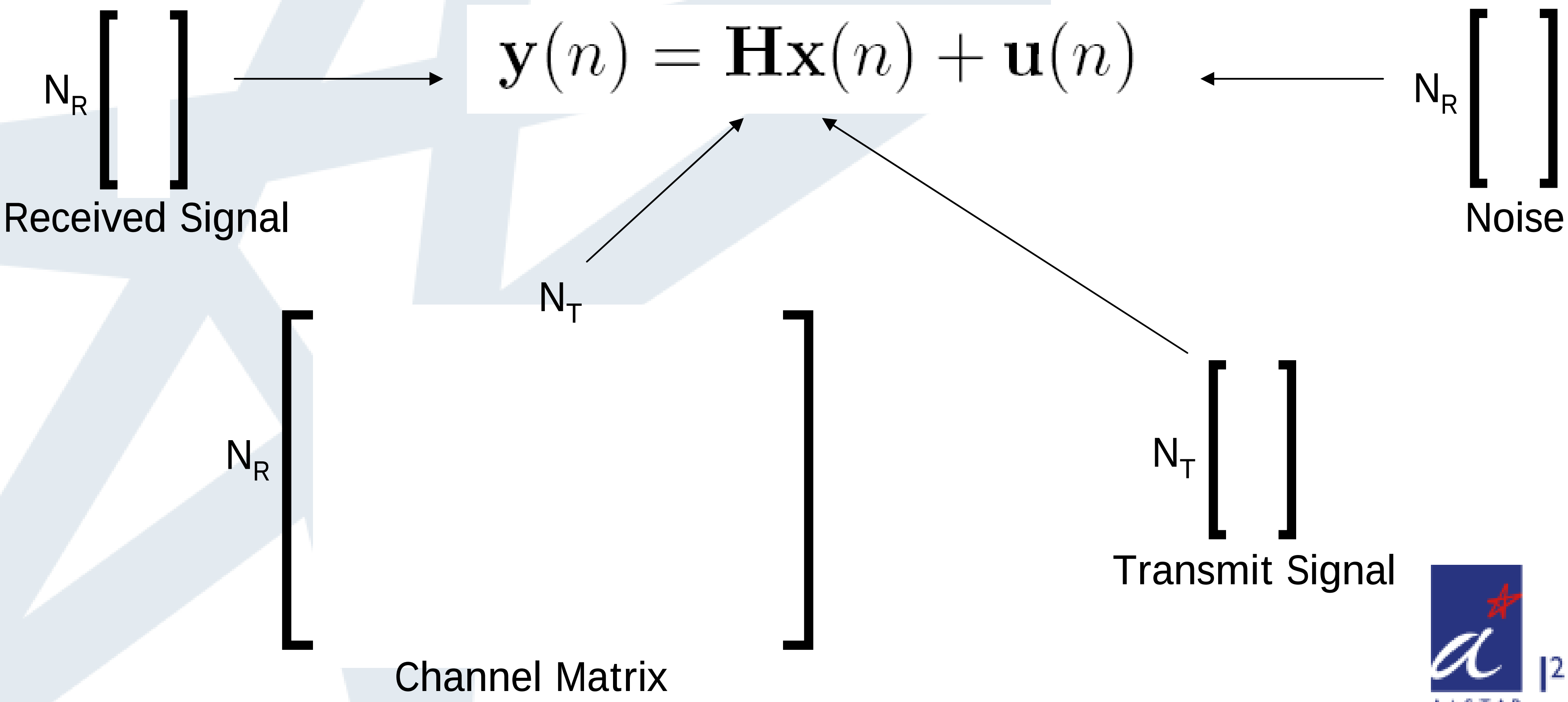
Single-user MIMO

- N_T transmitting antennas
- N_R receiving antennas

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{u}(n)$$



Single-user MIMO

Transmission Strategies

- Singular Value Decomposition (SVD) $\mathbf{H} = \mathbf{USV}^{\mathbf{H}}$
  - ➢ Different constellations for each subchannel
- Geometric Mean Decomposition (GMD)[1] $\mathbf{H} = \mathbf{QRP}^{\mathbf{H}}$
  - ➢ Same constellation for every subchannel
  - ➢ Reduce transceiver complexity

[1] Y. Jiang, J. Li and W. W. Hager, "Joint Transceiver Design for MIMO Communications Using Geometric Mean Decomposition,"
IEEE Trans. Signal Processing, vol. 53, no. 10, pp. 3791-3803, Oct. 2005.

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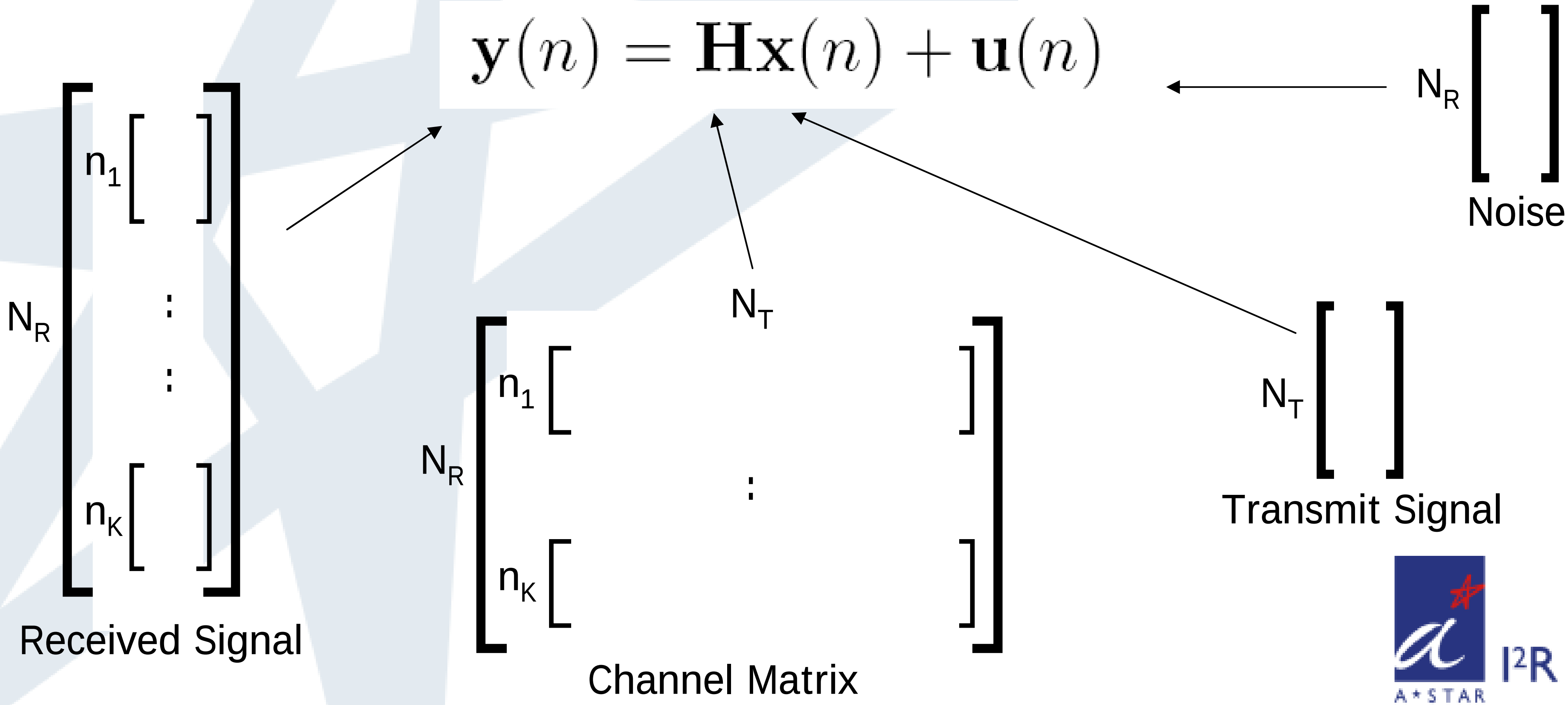
Multi-User Channel Model

- N_T transmitting antennas
- K users with $n_1, \ldots, n_K$ receiving antennas
- $N_R = n_1 + \ldots + n_K$

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{u}(n)$$



Problem Statement

- How to do Equal Rate Coding
- via New Matrix Decomposition
- to reduce transceiver complexity

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Block-Diagonal Geometric Mean Decomposition (BD-GMD)

Block-Diagonal GMD

$$H = P\,L\,Q^H$$

- P: Block Diagonal & Unitary
- L: Lower Triangular
- Q^H: Unitary

$$\begin{bmatrix} \mathbf{P}_1 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{P}_K \end{bmatrix}$$

Each P_i is unitary.

$$\begin{bmatrix} \mathbf{L}_1 & 0 & \dots & 0 \\ X & \mathbf{L}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X & X & \dots & \mathbf{L}_K \end{bmatrix}$$

Each L_i is equal diagonal.

Block-equal-diagonal

BD-GMD: Algorithm

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathcal{H} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathcal{P} \end{bmatrix} \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathfrak{L} & \mathcal{L} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1^H \\ \mathcal{Q}^H \end{bmatrix}$$

Expanding…

$$\mathbf{H}_1 = \mathbf{P}_1 \mathbf{L}_1 \mathbf{Q}_1^H,$$

$$\mathcal{H} = \mathcal{P} \mathfrak{L} \mathbf{Q}_1^H + \mathcal{P} \mathcal{L} \mathcal{Q}^H$$

GMD

From the 2nd equation…

$$\mathcal{H}(\mathbf{I} - \mathbf{Q}_1 \mathbf{Q}_1^H) = \mathcal{P} \mathcal{L} \mathcal{Q}^H$$

$$\mathfrak{L} = \mathcal{P}^H \mathcal{H} \mathbf{Q}_1$$

BD-GMD

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BD-GMD: Diagonal Elements

$$\begin{bmatrix} \widehat{\mathbf{H}}_i \\ \mathcal{H} \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{P}}_i & \mathbf{0} \\ \mathbf{0} & \mathcal{P} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{L}}_i & \mathbf{0} \\ \mathfrak{L} & \mathcal{L} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{Q}}_i^H \\ \mathcal{Q}^H \end{bmatrix}$$

$$\widehat{\mathbf{H}}_i = \widehat{\mathbf{P}}_i \widehat{\mathbf{L}}_i \widehat{\mathbf{Q}}_i^H$$

Taking determinants…

$$\det(\widehat{\mathbf{H}}_i \widehat{\mathbf{H}}_i^H) = \det(\widehat{\mathbf{L}}_i \widehat{\mathbf{L}}_i^H) = \prod_{j=1}^{i} r_j^{2n_j}$$

Therefore,

$$r_i = \sqrt[2n_i]{\frac{\det(\widehat{\mathbf{H}}_i \widehat{\mathbf{H}}_i^H)}{\det(\widehat{\mathbf{H}}_{i-1} \widehat{\mathbf{H}}_{i-1}^H)}}$$

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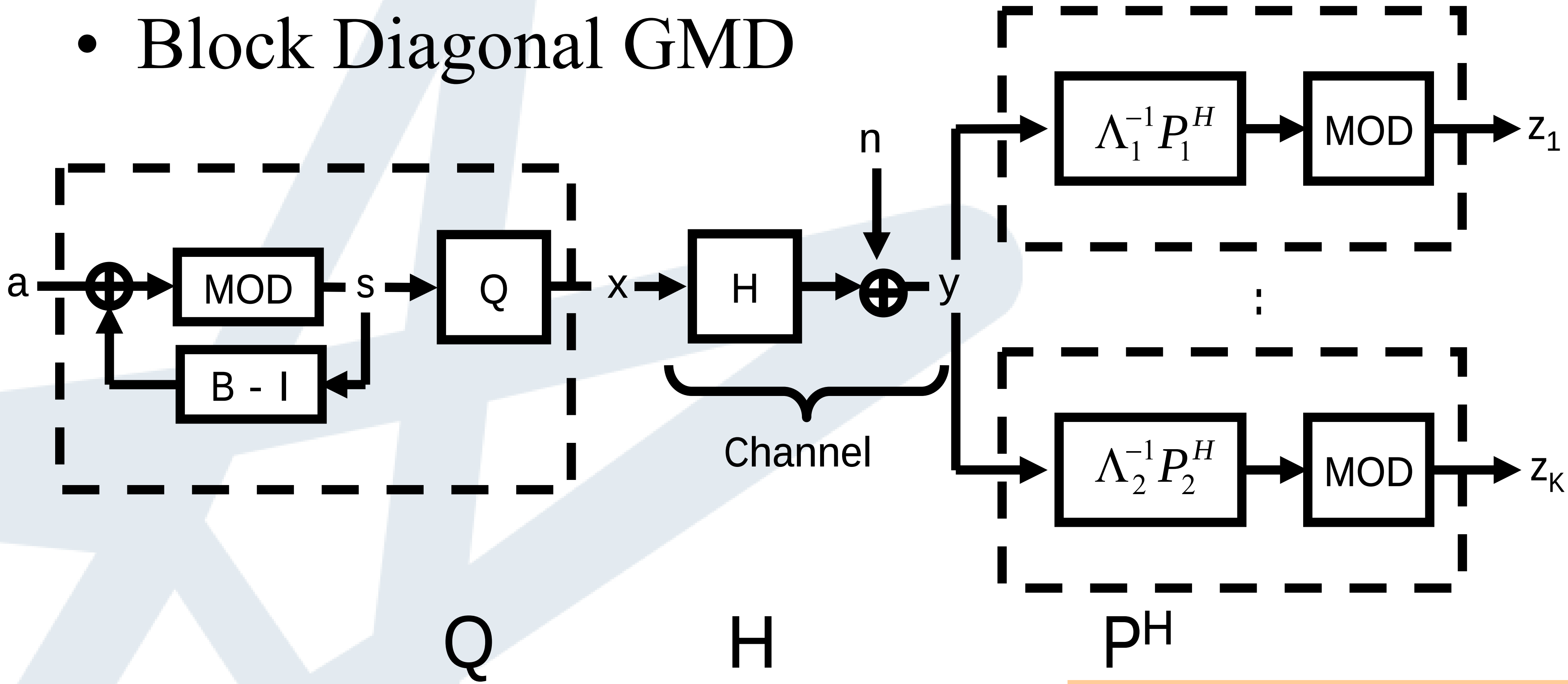
BD-GMD

- Block Diagonal GMD



Block-equal-rate

$L = \Lambda B$

$H = P\ L\ Q^H$ ← Unitary

P: Block Diagonal & Unitary

L: Lower Triangular

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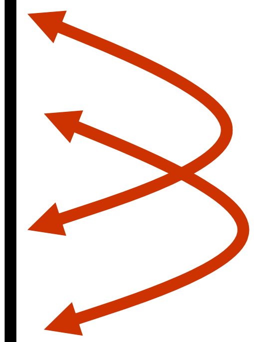
Considerations

- User Ordering
- Power loading
  - ➢ Equal
  - ➢ Channel inversion
    - ♦ Increase user fairness (Equal Rate)

User Ordering

- Rearranging the Channel Matrix

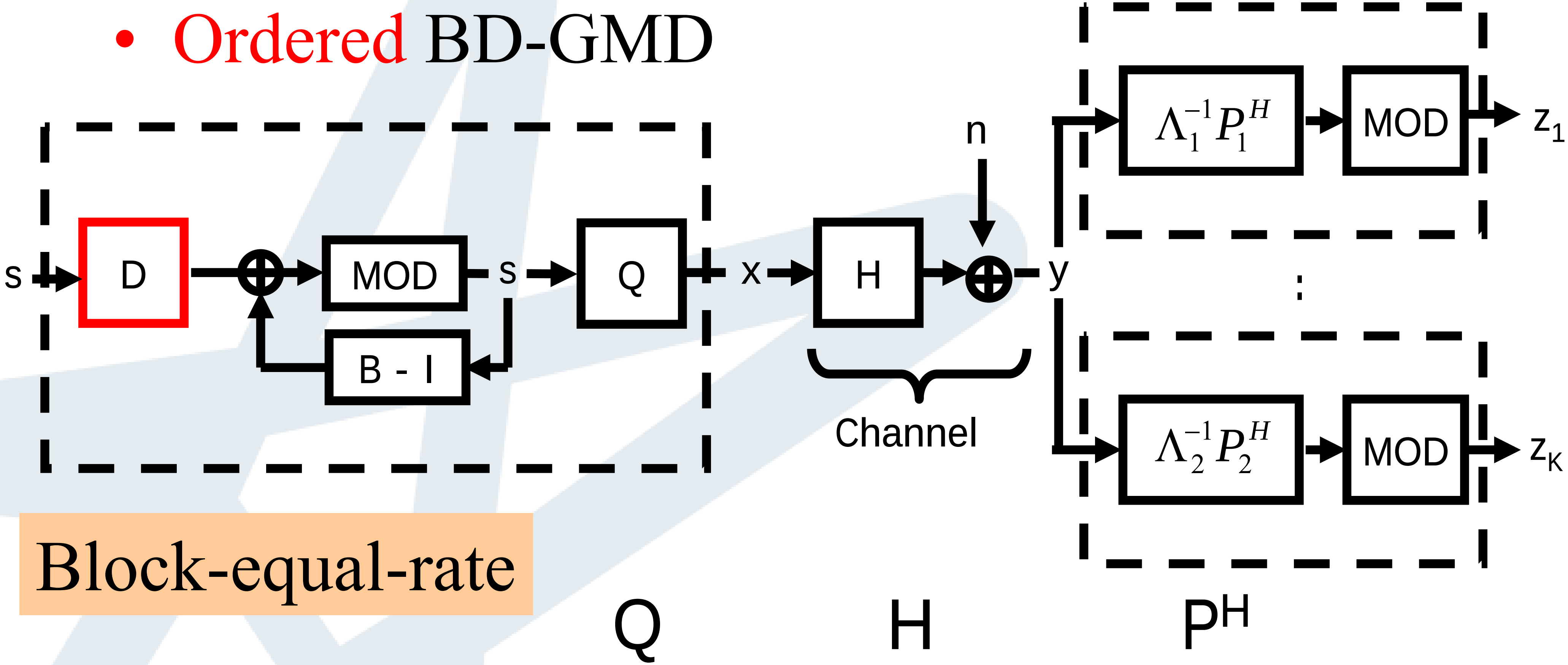
$$PLQ^H = H = N_R \overset{N_T}{\begin{bmatrix} n_1 \begin{bmatrix} \quad & \quad \end{bmatrix} \\ \vdots \\ n_K \begin{bmatrix} \quad & \quad \end{bmatrix} \end{bmatrix}}$$



to optimize diagonal elements in L.

User Ordering

- Ordered BD-GMD



Block-equal-rate

$L = \Lambda B$

$D H = P L Q^H$

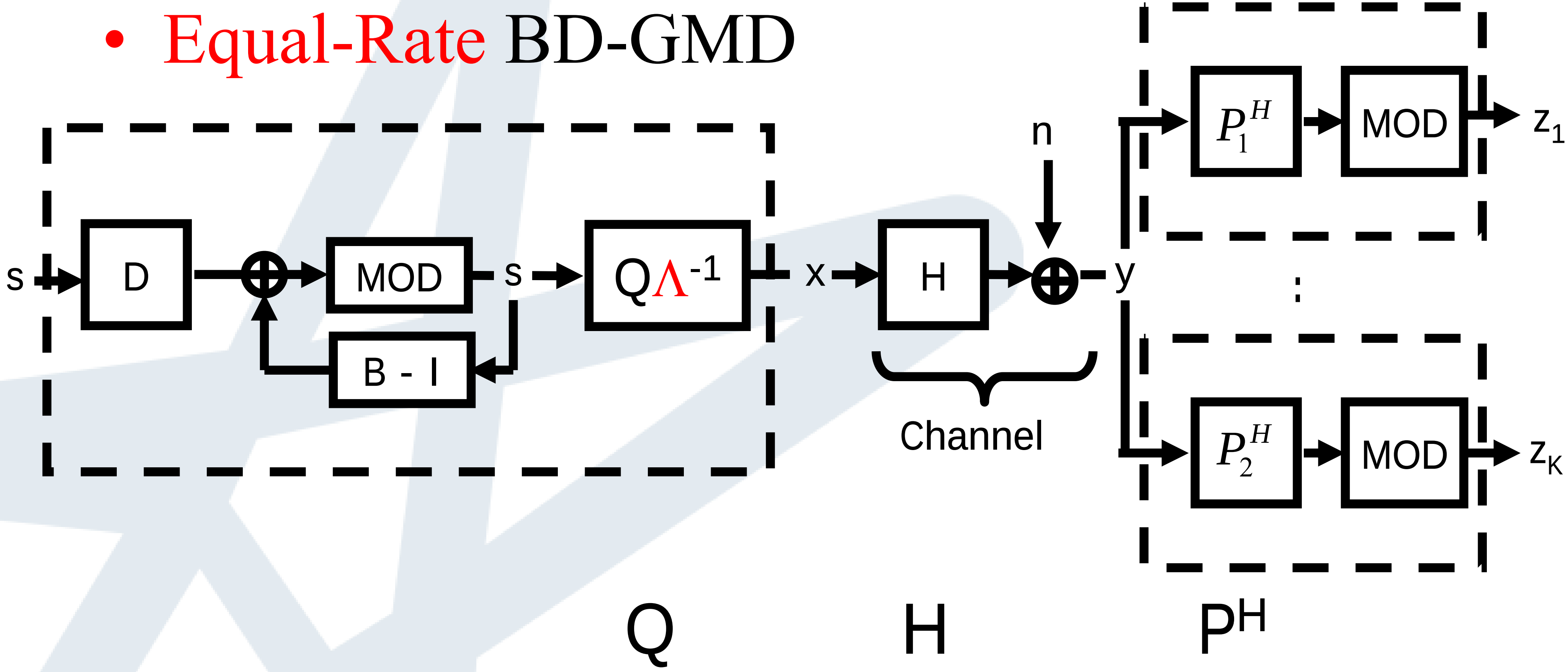
Unitary

Block Diagonal & Unitary

Lower Triangular

Power Loading

- Equal-Rate BD-GMD



$L = B\Lambda$

$D\,H = P\,L\,Q^H$

Unitary

Block Diagonal & Unitary

Lower Triangular

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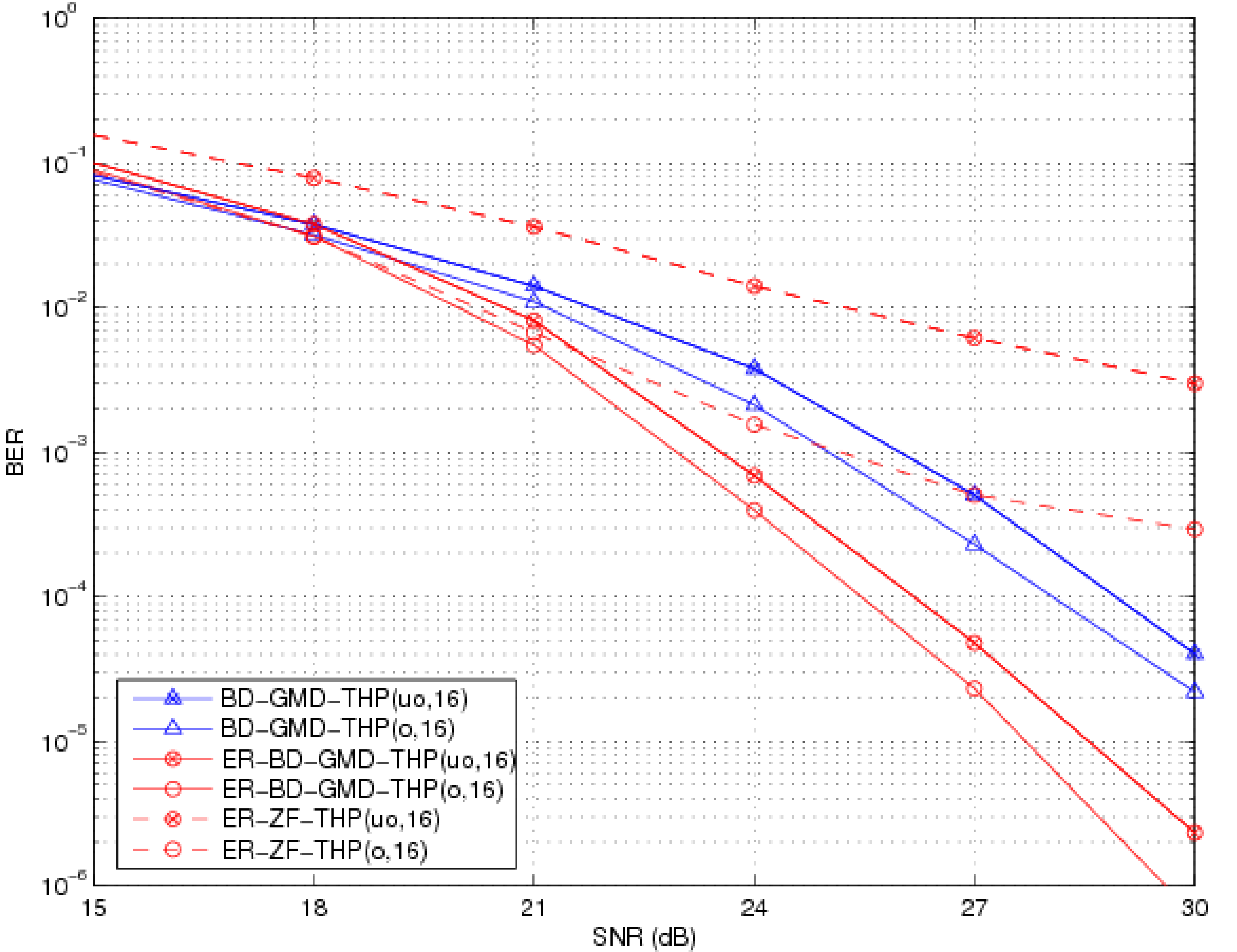
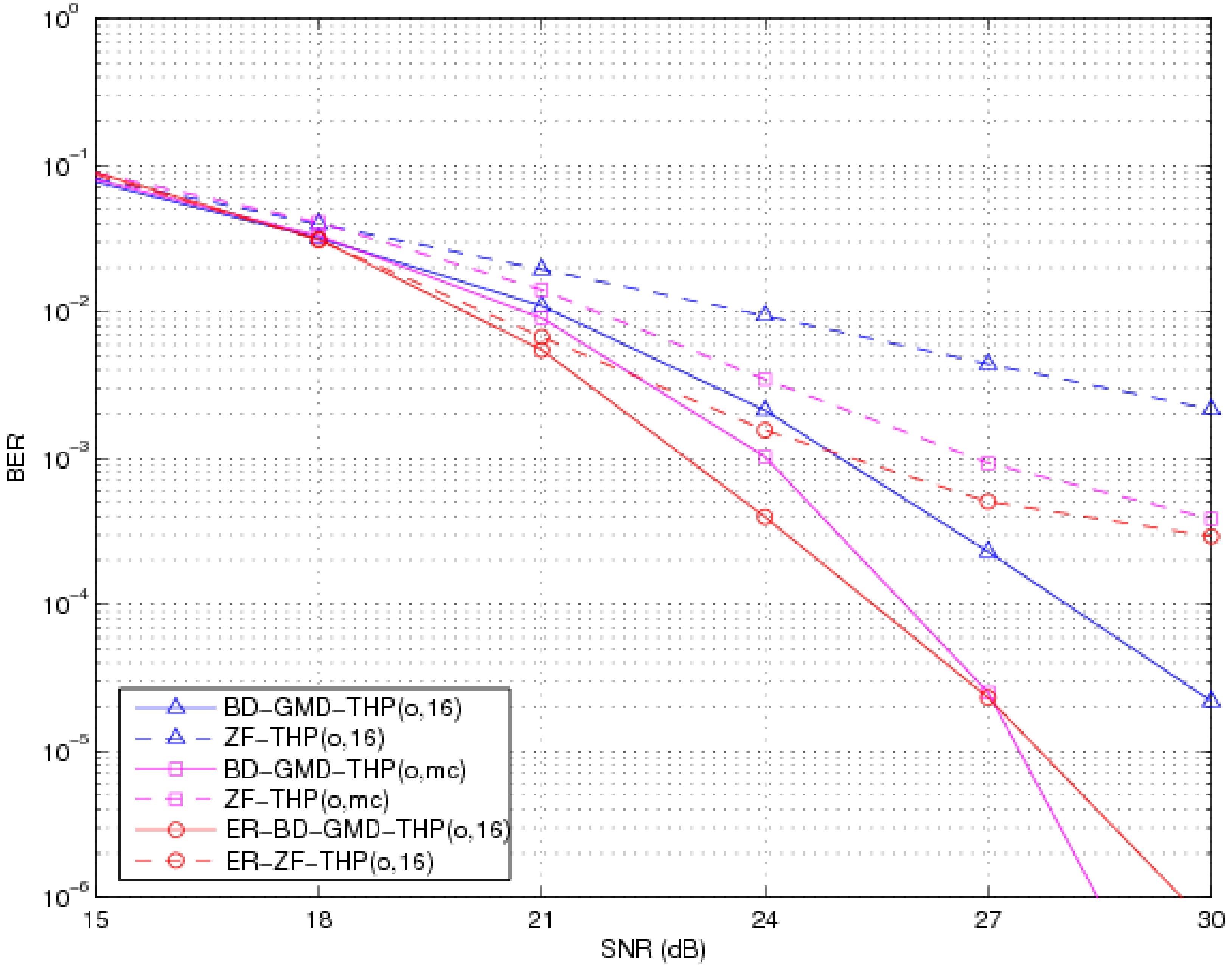
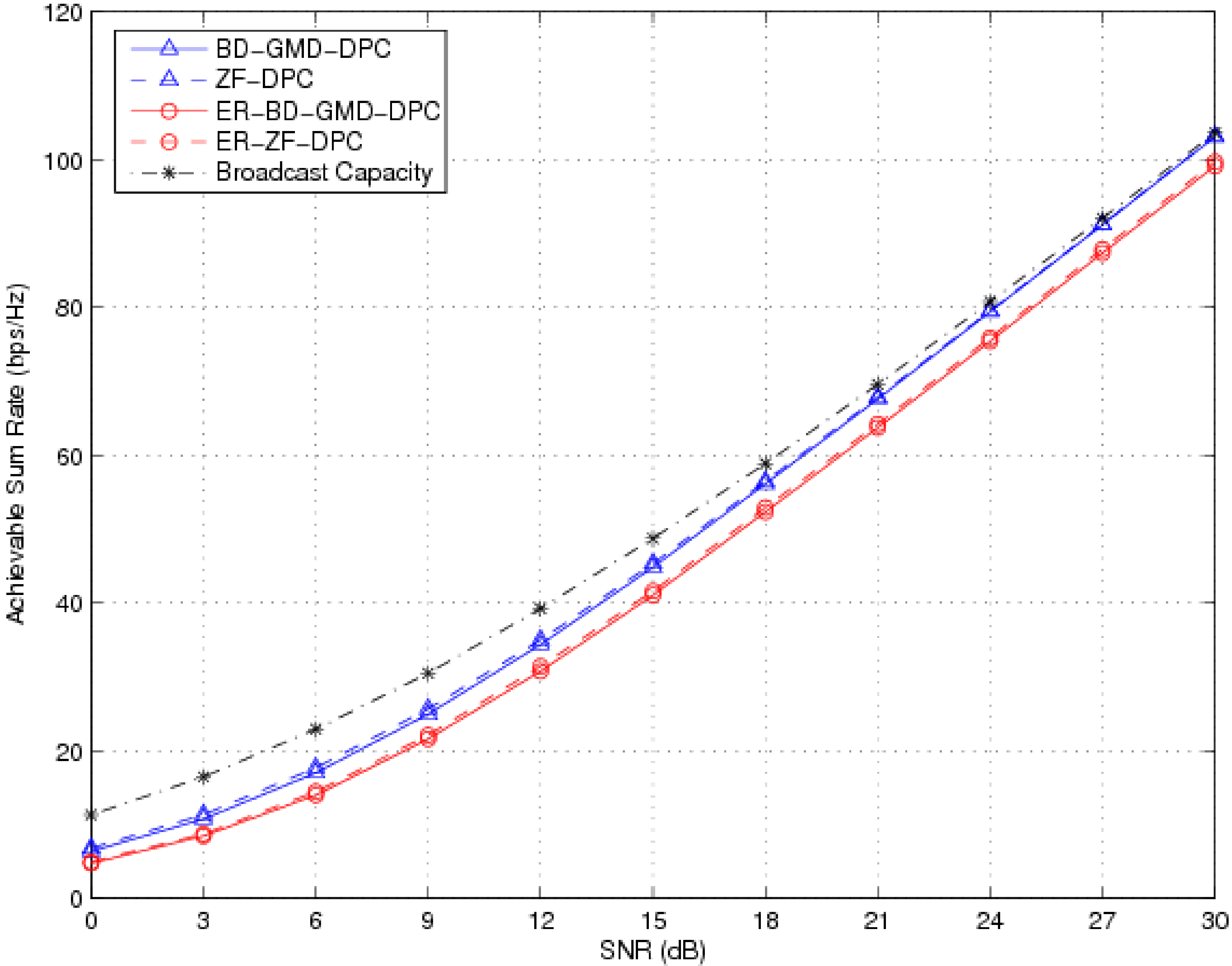
BER performance of ordered and unordered schemes



Effect of receiver equalization and multiple constellations on BER performance



Achievable sum rates of proposed schemes



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Conclusion

- Equal rate coding reduces transceiver complexity
- New matrix decomposition
- Improvements
  - User ordering
  - Power-loading

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