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MIMO BROADCAST COMMUNICATIONS USING BLOCK-DIAGONAL UNIFORM CHANNEL DECOMPOSITION (BD-UCD)

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ABSTRACT

Variable-rate coding for a multiple input multiple output (MIMO) channel increases transceiver complexity. In single user case, the uniform channel decomposition (UCD) scheme overcomes this problem by generating decoupled subchannels with identical SNRs so that equal-rate (ER) coding can be applied. In this paper, the solution is extended to the multiuser broadcast case. The block-diagonal geometric mean decomposition (BD-GMD) is used to design a capacity-achieving scheme, called the block-diagonal UCD (BD-UCD). It allows each user to apply ER coding on its own subchannels. An efficient near-optimal algorithm for multiuser uplink beamforming with SINR constraints is also proposed. Using this and duality, a scheme that allows ER coding to be applied to *every* subchannel of every user is designed. Simulations have shown that both schemes have superior BER performance and higher achievable sum-rates than conventional schemes.

I. INTRODUCTION

Multiple input multiple output (MIMO) systems have been studied extensively because of their exceptional power and bandwidth efficiencies [1]. In point-to-point communications, having channel state information (CSI) at the transmitter allows the use of the singular value decomposition (SVD) and water-filling in generating decoupled single input single output (SISO) subchannels with different signal-to-noise ratios (SNRs). Thus, variable-rate coding is usually employed among the data streams. This increases the transceiver complexity.

In [2], a capacity-achieving MMSE-based uniform channel decomposition (UCD) scheme that uses the geometric mean decomposition (GMD) was proposed. When used in conjunction with dirty paper coding (DPC) or a decision feedback equalization (DFE) receiver, subchannels with identical SNRs are achieved, thus allowing equal-rate codes to be applied.

These single-user equal-rate methods can be extended to MIMO broadcast channels. In [3], a new matrix decomposition, called the block-diagonal (BD-) GMD, was proposed. It was used to design a zero-forcing (ZF) based scheme that employs DPC at the transmitter and generates subchannels with identical SNRs for each user. This scheme was called the BD-GMD-DPC. Tomlinson-Harashima precoding (THP) can also be used as a simple suboptimal implementation of DPC. In the same paper, a scheme which generates subchannels with identical SNRs for *all* the users was also proposed. This scheme was called the equal-rate (ER-) BD-GMD-DPC. For convenience, we say that a scheme is *block-equal-rate* if each user enjoys identical SNRs for their own subchannels. If all the subchannels of all the users have identical SNRs, we say that the scheme is *equal-rate*.

In this paper, two more applications of the BD-GMD are considered. Firstly, the BD-GMD and uplink-downlink duality [4] [5] is used in designing a *capacity-achieving* block-equal-rate MMSE-based scheme called the block-diagonal (BD-) UCD-DPC. Jindal et al.'s sum-power iterative water-filling algorithm [9] is used to obtain the optimal transmit power allocation and precoder for the scheme. Secondly, an equal-rate near-capacity-achieving scheme called the equal-rate (ER-) BD-UCD-DPC is proposed. To do this, the problem of uplink beamforming under signal-to-interference-plus-noise-ratio (SINR) constraints [6] for mobile users with multiple antennas is first considered. An efficient algorithm which finds a near-optimal solution is then given, and duality is applied to produce the desired scheme. Simulations show that this scheme has an achievable sum-rate close to the broadcast capacity.

The paper is organized as follows. The MIMO broadcast channel model is presented in Section II, and the MMSE-DFE, MMSE-based DPC and BD-GMD are reviewed in Section III. Section IV details the application of the uplink-downlink duality result to the MIMO broadcast situation. In Sections V and VI, the BD-UCD-DPC and ER-BD-UCD-DPC schemes are designed. Simulation results are presented and discussed in Section VII, and a conclusion is given in Section VIII.

The following notations are used in the paper. The boldface is used to denote matrices and vectors, and $\mathrm{E}[\cdot]$ for expectation. Let $\mathrm{Tr}(\mathbf{X})$, $\mathbf{X}^T$, $\mathbf{X}^H$ and $\mathbf{X}^{-1}$ denote the matrix trace, transpose, conjugate transpose and inverse, respectively, for a matrix $\mathbf{X}$. $[\mathbf{X}]_{i,j}$ denotes the matrix element at the i-th row and j-th column. The diagonal matrix with elements $x_1, \ldots, x_n$ is denoted by $\mathrm{diag}(x_1, \ldots, x_n)$ while $\mathrm{diag}(\mathbf{X})$ is the diagonal matrix having the same diagonal as a matrix $\mathbf{X}$. $\|\cdot\|$ denotes the vector Euclidean norm. Let $|x|$ denote the absolute value, and x^* the conjugate, of a complex number x.

II. CHANNEL MODEL

Given an infrastructure based system with one base station (BS) and K mobile users, consider the *broadcast channel* from the BS to the mobile users. The BS is equipped with N_T antennas, and the i-th mobile user has n_i antennas. Let $N_R = \sum_{i=1}^{K} n_i$ be the total number of receive antennas. The input-output relation can be represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u}\,, \tag{1}$$

where $\mathbf{x}$ is the $N_T \times 1$ transmit signal vector at the BS, $\mathbf{y}$ the $N_R \times 1$ receive signal vector with $\mathbf{y} = [\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T]^T$, and

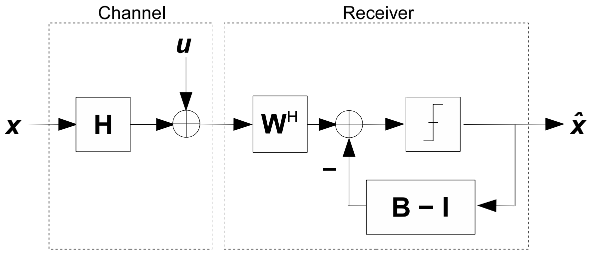

Figure 1: Block Diagram of MMSE-DFE.

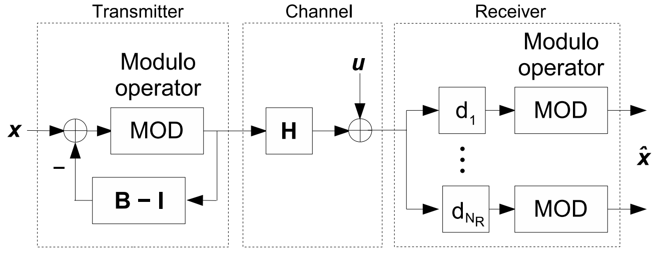

Figure 2: Block Diagram of THP.

each $\mathbf{y}_i$ the $n_i \times 1$ receive signal vector of user i. Assume that the noise vector $\mathbf{u}$ is zero-mean circularly symmetric complex Gaussian (CSCG) with $\mathrm{E}[\mathbf{u}\mathbf{u}^H] = N_0\mathbf{I}$, and $\mathbf{u}$ is independent of $\mathbf{x}$. Assume also that $\mathrm{E}[\|\mathbf{x}\|^2] = E_s$, and let $\rho = E_s/N_0$ be the SNR. Denote this channel by $N_T \times \{n_1, \ldots, n_K\}$.

If $\mathbf{x}$ is a Gaussian random vector, the sum-capacity of this broadcast channel [4], [5] is given by

$$\sup_{\mathrm{Tr}(\mathbf{F}\mathbf{F}^H)\leq E_s} \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{H}^H\mathbf{F}\mathbf{F}^H\mathbf{H}\right), \tag{2}$$

where $\mathbf{F}$ is of the block-diagonal form

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1 & 0 & \ldots & 0 \\ 0 & \mathbf{F}_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{F}_K \end{bmatrix} \tag{3}$$

and each block $\mathbf{F}_i$ is a $n_i \times n_i$ matrix. This sum-capacity can be achieved by a scheme that combines DPC with linear precoding. A fundamental step in proving this theorem is showing that there is a dual relationship between uplink DFE and downlink DPC schemes. This uplink-downlink duality result will be discussed in greater detail in Section IV.

III. Preliminaries

In this section, a review of two basic transceiver techniques is given, namely the MMSE-DPC and a general view of the MMSE-based DPC. The BD-GMD is also introduced.

A. MMSE-DFE

Consider the $N_T \times N_R$ point-to-point channel $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u}$ where $\mathrm{E}[\mathbf{x}\mathbf{x}^H] = (E_s/N_T)\mathbf{I}$ and $\mathrm{E}[\mathbf{u}\mathbf{u}^H] = N_0\mathbf{I}$. The MMSE-based DFE can be represented by the block diagram in Figure 1. Its nulling matrix is $\mathbf{W}^H = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \eta\mathbf{I})^{-1}$, where $\eta = N_0(N_T/E_s)$. It applies successive interference cancelation (SIC) via the feedback matrix $\mathbf{B} - \mathbf{I}$. Here, $\mathbf{B}$ is referred to as the interference matrix, and it satisfies $[\mathbf{B}]_{i,j} = [\mathbf{W}^H\mathbf{H}]_{i,j}$ if $i < j$, and $[\mathbf{B}]_{i,j} = [\mathbf{I}]_{i,j}$ otherwise. Denote this by $\mathbf{B} = \mathcal{U}(\mathbf{W}^H\mathbf{H})$. For any square matrix $\mathbf{X}$, also define $\mathcal{L}(\mathbf{X}) = \mathcal{U}(\mathbf{X}^H)^H$ for convenience.

Now, alternatively, the nulling and interference matrices can be found via the QR-decomposition [7]

$$\begin{bmatrix} \mathbf{H} \\ \sqrt{\eta}\mathbf{I} \end{bmatrix} = \mathbf{Q}_1\mathbf{R}_1 = \begin{bmatrix} \mathbf{Q}_{1u} \\ \mathbf{Q}_{1d} \end{bmatrix} \mathbf{\Lambda}_1\mathbf{B}_1 \tag{4}$$

where $\mathbf{Q}_1$ is unitary, $\mathbf{Q}_{1u}$ is $N_R \times N_T$, $\mathbf{Q}_{1d}$ is $N_T \times N_T$, $\mathbf{R}_1$ is a $N_T \times N_T$ upper triangular matrix, $\mathbf{\Lambda}_1 = \mathrm{diag}(\mathbf{R}_1)$, and $\mathbf{B}_1$ is upper triangular with unit diagonal. Then, the nulling and interference matrices satisfy $\mathbf{W}^H = \mathbf{\Lambda}_1^{-1}\mathbf{Q}_{1u}^H$ and $\mathbf{B} = \mathbf{B}_1$. Note that the MMSE-DFE is a capacity-achieving receiver.

B. MMSE-based DPC

One major problem with DFEs is error propagation. If CSI is known at the transmitter, interference between subchannels can be canceled completely at the transmitter via dirty paper coding (DPC). Here, a general view of MMSE-based DPC via successive interference pre-subtraction is developed.

Consider once again the $N_T \times N_R$ point-to-point channel $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u}$. However, it will not be required that $\mathrm{E}[\mathbf{u}\mathbf{u}^H] = N_0\mathbf{I}$ but only that $\mathrm{E}[|u_i|^2] = N_0$ for each i. Assume that there is no collaboration between the receive antennas. Let $h_{ij} = [\mathbf{H}]_{i,j}$. The i-th subchannel can be written as

$$y_i = (\sum_{j<i} h_{ij}x_j) + h_{ii}x_i + (\sum_{j>i} h_{ij}x_j) + u_i. \tag{5}$$

Suppose $(\sum_{j<i} h_{ij}x_j)$ can be canceled at the transmitter as interference terms. Then, SISO MMSE receivers that see $(\sum_{j>i} h_{ij}x_j) + u_i$ as noise terms can be used on each subchannel. The MMSE coefficient for the i-th subchannel is

$$d_i = \frac{h_{ii}^*}{\eta + \sum_{j\geq i} |h_{ij}|^2}. \tag{6}$$

Denoting $\mathbf{D}_d = \mathrm{diag}(d_1, \ldots, d_{N_R})$, the equivalent channel is now $\mathbf{D}_d\mathbf{H}$. The inference terms can now be representation by the interference matrix $\mathbf{B} = \mathcal{L}(\mathbf{D}_d\mathbf{H})$. Meanwhile, the SINR of the i-th subchannel is given by

$$\rho_i = \frac{|h_{ii}|^2}{\eta + \sum_{j>i} |h_{ij}|^2}. \tag{7}$$

A useful relation between (6) and (7) can be noted at this point. Let $\Sigma_0 = \eta + \sum_{j>i} |h_{ij}|^2$. Then, $\rho_i = |h_{ii}|^2/\Sigma_0$ and $d_i = h_{ii}^*/(\Sigma_0 + |h_{ii}|^2)$. Eliminating Σ_0 gives

$$d_i h_{ii} = \frac{\rho_i}{1 + \rho_i}. \tag{8}$$

One suboptimal implementation of DPC is Tomlinson Harashima precoding (THP) [8]. The block diagram of a MMSE-based DPC scheme using THP is shown in Figure 2. One downside of THP is the *precoding loss*, or slight increase in the average transmit power by a factor of $M/(M-1)$ for M-QAM symbols. For large constellations, this loss is negligible.

C. BD-GMD

In [3], it was shown that given a complex $N_R \times N_T$ matrix $\mathbf{H}$, and K positive integers $n_1, \ldots, n_K$ such that $N_R = \sum_{i=1}^{K} n_i$, there exists a decomposition

$$\mathbf{H} = \mathbf{PLQ}^H , \tag{9}$$

such that $\mathbf{Q}$ is unitary, $\mathbf{L}$ is lower triangular, and $\mathbf{P}$ is block diagonal of the form in (3) with each block $\mathbf{P}_i$ unitary. Furthermore, if we write $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \ldots, \mathbf{H}_n^T]^T$ where $\mathbf{H}_i$ has n_i rows, $\mathbf{Q} = [\mathbf{Q}_1, \mathbf{Q}_2, \ldots \mathbf{Q}_n]$ where $\mathbf{Q}_i$ has n_i columns and $\mathbf{L}_i$ as the i-th diagonal block of L, then $\mathbf{H}_i = \mathbf{P}_i\mathbf{L}_i\mathbf{Q}_i^H$ and this is the geometric mean decomposition (GMD) of $\mathbf{H}_i$.

IV. UPLINK-DOWNLINK DUALITY

We return to the $N_T \times \{n_1, \ldots, n_K\}$ MIMO broadcast channel described in Section II. The uplink-downlink duality results [4], [5] will now be used to construct a *dual* DPC scheme that consumes the same power and achieves the same rates as a given MMSE-DFE scheme.

First, suppose the following MMSE-DFE scheme is given. Consider the $\{n_1, \ldots, n_K\} \times N_T$ uplink channel $\mathbf{y} = \mathbf{H}^H\mathbf{x} + \mathbf{u}$ which has K mobile users with $n_1, \ldots, n_K$ transmit antennas respectively, and a BS with N_T receive antennas. Let $\mathrm{E}[\mathbf{uu}^H] = N_0\mathbf{I}$, and $\mathrm{E}[\|\mathbf{x}\|^2] = E_s$. This channel is dual to the broadcast channel in Section II. Meanwhile, let each user i be equipped with a pre-determined linear precoder $\mathbf{F}_i$. Combine all the precoders in a block-diagonal matrix $\mathbf{F}$ of the form (3), and consider a MMSE-DFE receiver at the BS. Using (4), the QR decomposition for the equivalent channel $\mathbf{H}^H\mathbf{F}$ is

$$\begin{bmatrix} \mathbf{H}^H\mathbf{F} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_u \\ \mathbf{Q}_d \end{bmatrix} \mathbf{\Lambda B} , \tag{10}$$

so the nulling and interference matrices are $\mathbf{W}^H = \mathbf{\Lambda}^{-1}\mathbf{Q}_u^H$ and $\mathbf{B}$ respectively. Write the i-th column of $\mathbf{F}$ as $\sqrt{p_i}\mathbf{f}_i$ where $\sqrt{p_i}$ is the norm and $\mathbf{f}_i$ a unit vector. Since $\mathbf{x} = \mathbf{Fs}$, p_i represents the power allocated to the i-th data symbol s_i. Thus, the total power is $\sum_{i=1}^{N_R} p_i = \mathrm{Tr}[\mathbf{FF}^H] = E_s$. Also, write the i-th column of $\mathbf{W}$ as $c_i\mathbf{w}_i$ where c_i is the norm and can be thought of as an MMSE weight. Assuming that the symbols are canceled perfectly in the SIC, the SINR of the i-th subchannel is

$$\rho_i = \frac{p_i|\mathbf{w}_i^H\mathbf{H}^H\mathbf{f}_i|^2}{N_0 + \sum_{j<i} p_j|\mathbf{w}_i^H\mathbf{H}^H\mathbf{f}_j|^2}. \tag{11}$$

Now, construct the dual DPC scheme for the broadcast channel as follows. Let $\widetilde{\mathbf{F}}$ be the linear precoder at the BS, $\widetilde{\mathbf{B}}$ the interference matrix, and $\widetilde{\mathbf{W}}^H$ the block-diagonal nulling matrix of the mobile users. The block diagram for this scheme is shown in Figure 3. First, define the i-th column of $\widetilde{\mathbf{F}}$ to be $\sqrt{q_i}\mathbf{w}_i$, where q_i is an unknown representing the power allocated to the i-th information symbol. Next, define the i-th column of $\widetilde{\mathbf{W}}$ to be $d_i\mathbf{f}_i$ where d_i is an unknown MMSE weight. Since the goal is to achieve the same SINRs, by (7), the power coefficients q_i need to satisfy

$$\rho_i = \frac{q_i|\mathbf{f}_i^H\mathbf{H}\mathbf{w}_i|^2}{N_0 + \sum_{j>i} q_j|\mathbf{f}_i^H\mathbf{H}\mathbf{w}_j|^2} \quad \text{for } 1 \le i \le N_R. \tag{12}$$

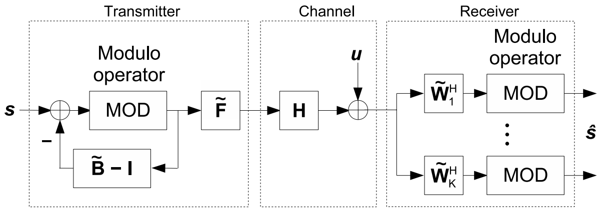

Figure 3: Block Diagram of Dual DPC Scheme.

Using the notations $\mathbf{q} = [q_1, \ldots, q_{N_R}]^T$, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_{N_R}]^T$ and $\alpha_{ij} = |\mathbf{f}_i^H\mathbf{H}\mathbf{w}_j|^2$, rewrite (12) in matrix form [2]:

$$\begin{bmatrix} \alpha_{11} & -\rho_1\alpha_{12} & \cdots & -\rho_1\alpha_{1N_R} \\ 0 & \alpha_{22} & \cdots & -\rho_2\alpha_{2N_R} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{N_RN_R} \end{bmatrix} \mathbf{q} = N_0\boldsymbol{\rho}. \tag{13}$$

Thus, $\mathbf{q}$ can be derived. In [4], the authors showed that $\sum_{i=1}^{N_R} q_i = \sum_{i=1}^{N_R} p_i$, so both the MMSE-DFE scheme and the dual DPC scheme consume the same total power E_s.

To compute the MMSE weights d_i, the relation in (8) gives $d_i\sqrt{q_i}(\mathbf{f}_i^H\mathbf{H}\mathbf{w}_i) = \rho_i/(1+\rho_i) = c_i\sqrt{p_i}(\mathbf{w}_i^H\mathbf{H}^H\mathbf{f}_i)$, which shows that $\mathbf{w}_i^H\mathbf{H}^H\mathbf{f}_i$ is real. Since $\mathbf{f}_i^H\mathbf{H}\mathbf{w}_i$ is its conjugate, they must be equal. Thus, $d_i\sqrt{q_i} = c_i\sqrt{p_i}$. Denote $\mathbf{D}_q = \mathrm{diag}(\sqrt{q_1}, \ldots, \sqrt{q_{N_R}})$ and similarly define diagonal matrices $\mathbf{D}_p$, $\mathbf{D}_c$ and $\mathbf{D}_d$ for $\{\sqrt{p_i}\}$, $\{c_i\}$ and $\{d_i\}$. Hence, $\mathbf{D}_d\mathbf{D}_q = \mathbf{D}_c\mathbf{D}_p$. Finally, to complete the dual DPC scheme, the interference matrix $\widetilde{\mathbf{B}}$ is computed. Using $\mathbf{B} = \mathcal{U}(\mathbf{W}^H\mathbf{H}^H\mathbf{F})$ and $\mathcal{L}(\mathbf{X})^H = \mathcal{U}(\mathbf{X}^H)$, one gets

$$\widetilde{\mathbf{B}} = \mathcal{L}(\widetilde{\mathbf{W}}^H\mathbf{H}\widetilde{\mathbf{F}}) = \mathbf{D}_c\mathbf{D}_q^{-1}\mathbf{B}^H\mathbf{D}_c^{-1}\mathbf{D}_q. \tag{14}$$

In [2], it was shown that $N_0(1+\rho_i) = \lambda_i^2$, where λ_i is the i-th diagonal element of $\mathbf{\Lambda}$. Thus, the achievable sum-rate for both the MMSE-DFE and dual DPC scheme can be written as

$$\sum_{i=1}^{N_R} \log(1+\rho_i) = \sum_{i=1}^{N_R} \log\left(\frac{\lambda_i^2}{N_0}\right) = \log\det(\mathbf{I} + \tfrac{1}{N_0}\mathbf{H}^H\mathbf{FF}^H\mathbf{H}). \tag{15}$$

V. BLOCK DIAGONAL UCD-DPC

In this section, a DPC scheme that is block-equal-rate and achieves the broadcast channel capacity is constructed. The idea is to first design a capacity-achieving MMSE-DFE scheme that generates subchannels with identical SNRs for each user by choosing an appropriate precoder $\mathbf{F}$. Then, the results from Section IV gives us a dual DPC scheme which is also capacity-achieving and has the same block-equal-rate property.

We begin with the dual uplink channel from Section IV. Let $\bar{\mathbf{F}}$ be a linear precoder for this uplink channel that achieves its sum-capacity, i.e. $\bar{\mathbf{F}}$ solves the optimization problem in (2). Different methods of solving this problem are described in [9], including Jindal et al.'s sum-power iterative water-filling algorithm which will be used in this paper. Note that for any block-diagonal unitary $\bar{\mathbf{P}}$, the precoder $\bar{\mathbf{F}}\bar{\mathbf{P}}$ gives the same sum-capacity as $\bar{\mathbf{F}}$. It remains for us to choose $\bar{\mathbf{P}}$ such that the MMSE-DFE scheme using the precoder $\mathbf{F} = \bar{\mathbf{F}}\bar{\mathbf{P}}$ is block-equal-rate. From (4), this is equivalent to finding $\bar{\mathbf{P}}$ such that the QR decomposition

$$\begin{bmatrix} \mathbf{H}^H\bar{\mathbf{F}}\bar{\mathbf{P}} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_u \\ \mathbf{Q}_d \end{bmatrix} \mathbf{\Lambda}\mathbf{B} \quad (16)$$

gives a $\mathbf{\Lambda}$ whose diagonal elements are equal in blocks of $n_1, \ldots, n_K$ elements respectively. Following [2], rewrite the LHS of (16) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{P}}^H \end{bmatrix} \begin{bmatrix} \mathbf{H}^H\bar{\mathbf{F}} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix} \bar{\mathbf{P}} \, . \quad (17)$$

Consider the BD-GMD of the middle term

$$\begin{bmatrix} \mathbf{H}^H\bar{\mathbf{F}} \\ \sqrt{N_0}\mathbf{I} \end{bmatrix}^H = \mathbf{P}\mathbf{L}\mathbf{Q}^H \, , \quad (18)$$

where $\mathbf{P}$ is block-diagonal, $\mathbf{L}$ is lower triangular, and both $\mathbf{P}$ and $\mathbf{Q}$ are unitary. Consequently, (16) becomes

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{P}}^H \end{bmatrix} \mathbf{Q}\mathbf{L}^H\mathbf{P}^H\bar{\mathbf{P}} = \begin{bmatrix} \mathbf{Q}_u \\ \mathbf{Q}_d \end{bmatrix} \mathbf{\Lambda}\mathbf{B} \, . \quad (19)$$

Hence, we can choose $\bar{\mathbf{P}} = \mathbf{P}$, $\mathbf{\Lambda}\mathbf{B} = \mathbf{L}^H$ and $\mathbf{Q}_u$ to be the top N_T rows of $\mathbf{Q}$. This gives us our desired capacity-achieving block-equal-rate MMSE-DFE scheme.

The dual DPC scheme that is block-equal-rate and capacity-achieving can now be constructed. The block diagram of this scheme using THP is shown in Figure 3. Section IV gives the precoding, interference and nulling matrices as

$$\widetilde{\mathbf{F}} = \mathbf{Q}_u\mathbf{\Lambda}^{-1}\mathbf{D}_c^{-1}\mathbf{D}_q \quad (20)$$

$$\widetilde{\mathbf{B}} = \mathbf{D}_c\mathbf{D}_q^{-1}\mathbf{L}\mathbf{\Lambda}^{-1}\mathbf{D}_c^{-1}\mathbf{D}_q \quad (21)$$

$$\widetilde{\mathbf{W}} = \bar{\mathbf{F}}\mathbf{P}\mathbf{D}_q^{-1}\mathbf{D}_c \, , \quad (22)$$

where $\mathbf{D}_q$ is calculated from (13) and $\mathbf{D}_c$ contains the column norms of $\mathbf{Q}_u\mathbf{\Lambda}^{-1}$. Call this scheme the block-diagonal (BD-) UCD-DPC, since the special single-user case of $K = 1$ is precisely the UCD-DPC scheme in [2].

VI. Equal-Rate BD-UCD-DPC

Sometimes, it is desirable to treat the mobile users fairly by providing every user with the same rate. In this section, a near-optimal DPC scheme for the broadcast channel that generates decoupled subchannels all with identical SNRs will be constructed. This construction can be generalized easily to other rate constraints for the users.

The crux lies in choosing the right precoder $\bar{\mathbf{F}}$ so that the method in Section V produces the desired equal-rate scheme. The resulting scheme will be called the *equal-rate* (ER-) BD-UCD-DPC. For the rest of this section, the focus will be on finding this precoder. Let $\mathbf{H} = [\mathbf{H}_1^T, \ldots, \mathbf{H}_K^T]^T$ where each $\mathbf{H}_i$ has n_i rows. Let $\bar{\mathbf{F}}_i$ be the i-th block of the block-diagonal $\bar{\mathbf{F}}$. Then, the rate of user i is [5]

$$R_i = \log \frac{\det(\mathbf{I} + \frac{1}{N_0}\sum_{j\le i}\mathbf{H}_j^H\bar{\mathbf{F}}_j\bar{\mathbf{F}}_j^H\mathbf{H}_j)}{\det(\mathbf{I} + \frac{1}{N_0}\sum_{j<i}\mathbf{H}_j^H\bar{\mathbf{F}}_j\bar{\mathbf{F}}_j^H\mathbf{H}_j)} . \quad (23)$$

Ideally, for each i, $R_i = n_i\bar{R}$ for some $\bar{R}$, and $\mathrm{Tr}(\bar{\mathbf{F}}\bar{\mathbf{F}}^H) \le Es$. The goal is to find $\bar{\mathbf{F}}$ such that $\bar{R}$ is maximized. It is unclear at this moment how this problem can be solved exactly. Meanwhile, a near-optimal algorithm of lower complexity inspired by [6] and [9] is proposed below.

The basic building block of the algorithm is as follows: given a target rate $\bar{R}$, find a precoder $\bar{\mathbf{F}}$ that achieves the rate $\bar{R}$ for every subchannel with minimum power. Rewrite (23) as

$$n_i\bar{R} = \log\det(\mathbf{I} + \tfrac{1}{N_0}\mathbf{G}_i\bar{\mathbf{F}}_i\bar{\mathbf{F}}_i^H\mathbf{G}_i^H) \, , \quad (24)$$

where $\mathbf{G}_i = (\mathbf{I} + \frac{1}{N_0}\sum_{j<i}\mathbf{H}_j^H\bar{\mathbf{F}}_j\bar{\mathbf{F}}_j^H\mathbf{H}_j)^{-1/2}\mathbf{H}_i^H$, using a trick from [9]. Thus, if $\mathbf{G}_i$ is given, then the minimum power $\bar{\mathbf{F}}_i$ satisfying (24) can be found by water-filling. Since $\mathbf{G}_i$ depends only on $\bar{\mathbf{F}}_j$ for $j < i$, equation (24) can be solved successively from $i = 1$ to $i = K$. Of course, to find the $\bar{\mathbf{F}}$ with minimum power satisfying the equations in (24) for all i, the $\bar{\mathbf{F}}_i$'s may need to be optimized jointly using iterative methods. However, to avoid incurring a high complexity cost, the above non-iterative algorithm will suffice for now.

Let $\mathrm{P}(\bar{R})$ be the power $\mathrm{Tr}(\bar{\mathbf{F}}\bar{\mathbf{F}}^H)$ of the precoder $\bar{\mathbf{F}}$ computed by the above algorithm for a target rate $(\bar{R})$. The algorithm can now be completed by iteratively finding $\bar{R}$ such that $\mathrm{P}(\bar{R}) = E_s$. Since there is a near-linear relation between $\bar{R}$ and $\log \mathrm{P}(\bar{R})$, a simple numerical method like the secant method can be used. Simulations show that convergence is typically achieved in less than six iterations.

VII. Simulations Results

In this section, computer simulation results are presented to evaluate the performance of the two schemes proposed in this paper: the BD-UCD-DPC, and the ER-BD-UCD-DPC. Firstly, the BD-UCD-DPC scheme is compared with the conventional multi-user MMSE-DPC scheme [8] which assumes no equalization on the side of the mobile users. This MMSE-DPC scheme can be viewed as a special case of the BD-UCD-DPC when $n_i = 1$ for all i. Secondly, the improvement of the two new MMSE-based schemes over their ZF-based counterparts in [3] is studied. Thirdly, the loss in performance of the ER-BD-UCD-DPC below the BD-UCD-DPC because of the additional equal-rate requirement is examined.

In the simulations, the $12 \times \{4, 4, 4\}$ broadcast channel is considered. The elements of the channel matrix $\mathbf{H}$ are assumed to be independent and CSCG with zero mean and unit variance. To compute the achievable sum-rates of the different schemes, perfect dirty paper coding (DPC) and Gaussian input at the transmitter are assumed. The results are based on 2000 Monte Carlo realizations of $\mathbf{H}$. To compute the BER curves, 16-QAM modulation is used, and THP is applied for interference presubtraction at the transmitter. The transmit power is scaled down by a factor of $15/16$ to account for the THP precoding loss. In the figures, the solid lines represent the new schemes, the triangles represent block-equal-rate schemes, while the circles represent equal-rate ones.

In Figure 4, note that there is only a tiny improvement in achievable sum-rate of the BD-UCD-DPC over the MMSE-DPC. This can be understood by studying the effect of pre-

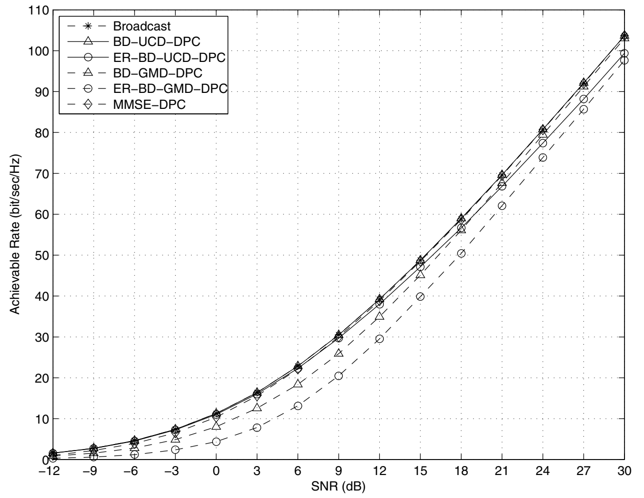

Figure 4: Achievable sum-rates of multi-user schemes.

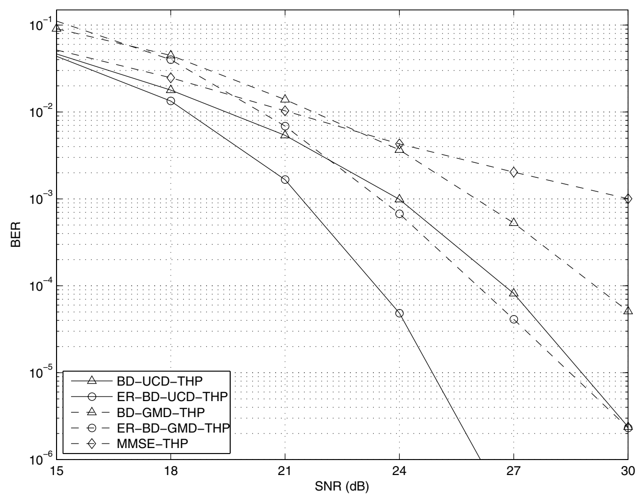

Figure 5: BER of multi-user schemes.

coding on the dual uplink channel capacity. The power loading aspect of precoding has a much greater effect on capacity than the beam-forming aspect. Thus, in the dual broadcast case, although MMSE-DPC does not enjoy equalization at the receivers, it does not suffer any significant loss in capacity. However, in Figure 5, the BD-UCD-THP shows a dramatic improvement in BER performance over the MMSE-THP, with more than 6 dB gain at BER of 10^{-3}. This is because of the diversity gain afforded by linear equalization at the receivers.

Now, the advantage of using a MMSE-based scheme against its ZF-based counterpart is shown. The BD-UCD-DPC enjoys a slight 1.5 dB gain in achievable sum-rate over the BD-GMD-DPC at the low SNR region. As expected, their performance converges with increasing SNR. Similarly, the achievable sum-rates of the ER-BD-GMD-DPC and ER-BD-UCD-DPC converge at high SNR. In terms of BER performance, the effect of error minimization is seen more clearly. In Figure 5, the BD-UCD-THP consistently shows a 3 dB gain in BER performance over the BD-GMD-THP at all SNRs. Meanwhile, the ER-BD-UCD-THP shows an improvement of as much as 4.5 dB over the ER-BD-GMD-THP at BER of 10^{-6}.

Finally, the feasibility of providing equal rates for all users is studied. In Figure 4, it is seen that the ER-BD-UCD-DPC achieves the same sum-rates as the BD-UCD-DPC at low SNR. Capacity in this SNR region is affected more by noise than by equal-rate constraints. At high SNR, the ER-BD-UCD-DPC only suffers a 1.5 dB loss. This also shows that the near-optimal iterative beamforming algorithm in Section VI does not experience much performance loss. In Figure 5, the ER-BD-UCD-THP demonstrates its superior BER performance over the BD-UCD-THP, with about 6 dB gain at BER of 10^{-5}. This is because its worst subchannel, which has a large influence on the average BER, is greatly elevated by the equal-rate constraint.

VIII. Conclusion

The two MMSE-based schemes, BD-UCD-DPC and ER-BD-UCD-DPC, for the MIMO broadcast channel have been presented. Both schemes use the BD-GMD and apply THP at the transmitter for interference pre-subtraction. The first scheme allows equal-rate coding for the subchannels of each user, while the second scheme allows equal-rate coding for every subchannel of every user. A near-optimal but efficient algorithm to solve the problem of beamforming under SINR constraints is proposed in the design of the second scheme. Simulations have shown that both schemes have better BER performance and higher achievable sum-rates than existing multiuser schemes such as the MMSE-DPC, BD-GMD-DPC and ER-BD-GMD-DPC.

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