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BLOCK-DIAGONAL GEOMETRIC MEAN DECOMPOSITION (BD-GMD) FOR MULTIUSER MIMO BROADCAST CHANNELS

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ABSTRACT

Matrix decompositions play an important role in analyzing the capacity and designing the transceiver for multiple input multiple output (MIMO) channels. In the single user case, by relying on the decision feedback equalizer (DFE) at the receiver or Tomlinson-Harashima precoding (THP) at the transmitter, the geometric mean decomposition (GMD) can be used to create identical signal-to-noise ratios (SNR) for each decoupled subchannel. In this paper, we propose a new matrix decomposition, called the block-diagonal GMD (BD-GMD), for the multiuser MIMO broadcast channel. Applying THP at transmitter and linear equalization in each of the receivers, each user can achieve identical SNRs for its subchannels, thus equal-rate coding can be applied for each user. Furthermore, by using transmit power control and the BD-GMD, we design a scheme that achieves equal-rate coding for the subchannels of all users. Computer simulations have shown that the proposed schemes have better BER performances than zero-forcing THP (ZF-THP) and equal-rate ZF-THP schemes.

I. INTRODUCTION

Wireless transmissions via multiple input multiple output (MIMO) antennas have received considerable attention during the past decade due to the promising capacity gain achieved through MIMO channels without relying on power increase or bandwidth expansion [1]. For single user MIMO, when channel state information (CSI) is known, the singular value decomposition (SVD) can be used to decompose the MIMO channel into a set of single input single output (SISO) subchannels, for which water-filling can be used to maximize the channel capacity. As each subchannel has a different signal-to-noise ratio (SNR) value, variable-rate coding is usually used among the data streams, which increases the transceiver complexity.

In [4], the authors proposed a geometric mean decomposition (GMD) for the channel matrix of point-to-point MIMO. Using GMD, a scheme combining linear precoding and nonlinear DFE receivers achieves identical SNR for all subchannels. Alternatively, dirty paper coding (DPC) can be applied with GMD to pre-subtract the interference before transmission. THP, a simple suboptimal implementation of DPC which increases transmit power slightly, can also be used [5]. For both DFE and DPC, equal-rate (ER) codes can be applied to all subchannels.

In this paper, we consider the multiuser MIMO broadcast channel, and propose a new matrix decomposition, called block-diagonal geometric mean decomposition (BD-GMD). Two applications of the proposed matrix decomposition are considered. In the first application, BD-GMD is combined with DPC to give a scheme in which the data streams of each user can be allocated with equal-rate codes. In the second application, BD-GMD is combined with DPC as well as transmit power control so that equal-rate codes can be applied to the subchannels for *all* the users.

This paper is organized as follows. The multiuser MIMO downlink channel model is presented in Section II, where the conventional schemes using THP are also reviewed. The mathematical formulation and algorithm of BD-GMD is presented in Section III. The BD-GMD-DPC and ER-BD-GMD-DPC transceiver designs are described in Sections IV and V respectively. Subsequently, simulations in Section VI compare these algorithms with ZF-THP. Finally, the conclusions are drawn in Section VII.

The following notations are used. The boldface is used to denote matrices and vectors. Let $\mathrm{Tr}(\mathbf{S})$, $\mathbf{S}^T$, $\mathbf{S}^H$ and $\mathbf{S}^{-1}$ denote the matrix trace, transpose, conjugate transpose and inverse, respectively, for a matrix, $\mathbf{S}$. $\|\cdot\|$ denotes the vector Euclidean norm, and $\mathrm{E}[\cdot]$ the expectation operator. $\mathrm{diag}(\mathbf{S})$ is the diagonal matrix with the diagonal elements of $\mathbf{S}$.

II. MULTIUSER COMMUNICATIONS

A. Channel Model

Let us consider an infrastructure based system with one base station (BS) and K mobile users. The BS is equipped with N_T antennas, and the mobile users have $n_1, n_2, \ldots, n_K$ antennas respectively. Let this system be denoted by $N_T \times \{n_1, n_2, \ldots, n_K\}$. We are now interested in the *broadcast channel* from the BS to the K users. Let $N_R = n_1 + n_2 + \ldots + n_K$ be the total number of receiver antennas. Denote $\mathbf{x}$ as the $N_T \times 1$ transmitted signal vector at the BS; $\mathbf{y}$ the $N_R \times 1$ received signal vector with $\mathbf{y} = [\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T]^T$, where $\mathbf{y}_i$, of dimension $n_i \times 1$, is the received signal vector of user k. The input-output relation can be represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{u}. \tag{1}$$

The noise vector $\mathbf{u}$ is assumed to be a zero-mean, circularly symmetric complex Gaussian (CSCG) vector with $E[\mathbf{u}\mathbf{u}^H] = N_0\mathbf{I}$, and $\mathbf{u}$ is independent of $\mathbf{x}$. Assume also that the total transmit power is $E_s = \mathrm{E}[\|\mathbf{x}\|^2]$, and let $\rho = E_s/N_0$ be the SNR.

In the downlink broadcast channel, there is usually no collaboration between the mobile users, so the users do not have signal information from the antennas belonging to other users. This makes interference cancelation and equalization at the receiver side difficult. Thus, it is important to have CSI at the

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transmitter, so that interference cancelation via dirty paper precoding can be performed.

Conventional precoding schemes, such as zero-forcing THP (ZF-THP) [5] and "novel" THP (N-THP) [3], treat multiple antennas of different users as different virtual users. The first, ZF-THP, is based on the QR decomposition $\mathbf{H}^H = \mathbf{QR}$, or $\mathbf{H} = \mathbf{R}^H\mathbf{Q}^H$. The linear precoder $\mathbf{Q}$ is applied before transmission so that $\mathbf{x} = \mathbf{Qs}$, where $\mathbf{s}$ is the vector of symbols to be sent. This transforms the channel to $\mathbf{y} = \mathbf{R}^H\mathbf{s} + \mathbf{u}$, on which THP is applied to pre-subtract the interference represented by the lower triangular matrix $\mathbf{R}^H$. Thus, we have decoupled subchannels, $y_i = r_i s_i + u_i$, where r_i is the ith diagonal element of $\mathbf{R}$.

In N-THP [3], the authors considered precoding matrices $\mathbf{F}$ such that the resulting channel matrix $\mathbf{HF}$ is lower triangular and equal diagonal. Here, $\mathbf{F}$ need not be unitary but only has to satisfy the power constraint $\mathrm{Tr}(\mathbf{F}^H\mathbf{F}) \leq E_s$. The precoder $\mathbf{F}$ that maximizes the equal diagonal element r in $\mathbf{HF}$ can be found algorithmically. The scheme now only needs to perform THP to cancel the interference represented by the lower triangular matrix $\mathbf{HF}$ before precoding with $\mathbf{F}$ and transmitting the signal. This scheme gives independent subchannels $y_i = rs_i + u_i$ on which equal-rate codes can be applied. Hence, we shall refer to the N-THP scheme as equal-rate ZF-THP (ER-ZF-THP).

III. Block-Diagonal GMD

Both ZF-THP and ER-ZF-THP assumes no collaboration between the receive antennas of each user. In the case where some of the mobile users have multiple antennas, the performance gain can be expected by doing equalization on the receiver side. This equalization can only be done for the data streams of the same user, but not among the users. We can represent it as a premultiplication of the receive signal $\mathbf{y}(n)$ by a matrix $\mathbf{A}$ of the block-diagonal form

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & \dots & 0 \\ 0 & \mathbf{A}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{A}_K \end{bmatrix}, \tag{2}$$

where each $\mathbf{A}_i$ is the $n_i \times n_i$ equalization matrix of user i. Each row of $\mathbf{A}_i$ is required to be of unit norm so that the noise vector $\mathbf{u}$ is not amplified by $\mathbf{A}$. Note that the situation where the mobile users have single antennas is represented by the case $\mathbf{A} = \mathbf{I}$. The matrix $\mathbf{A}$ gives us the equivalent channel matrix $\mathbf{AH}$ which can then be canceled by transmitter techniques such as ZF-THP or ER-ZF-THP.

Consider the QR-decomposition $\mathbf{AH} = \mathbf{R}^H\mathbf{Q}^H$. Since the aim is to let each user have equal rates for its data streams, it is natural to ask if it can be accomplished by choosing an appropriate equalization matrix $\mathbf{A}$. To simplify this problem, assume further that $\mathbf{A}$ is unitary. Our problem can now be stated as follows.

A. *Mathematical Formulation*

Let $\mathbf{H}$ be an $N_R \times N_T$ matrix, and $n_1, ..., n_K$ a sequence of integers such that $N_R = n_1 + n_2 + \ldots + n_K$. We want to find a matrix decomposition

$$\mathbf{H} = \mathbf{PLQ}^H, \tag{3}$$

such that $\mathbf{Q}$ is a unitary $N_R \times N_T$ matrix and $\mathbf{P}$ is a block-diagonal matrix of the form in (2) where each block $\mathbf{P}_i$ is a unitary $n_i \times n_i$ matrix. $\mathbf{L}$ is a lower triangular matrix, whose diagonal elements are equal in blocks of $n_1, ..., n_K$ elements respectively.

B. *Algorithm*

Write the product $\mathbf{H} = \mathbf{PLQ}^H$ as

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathcal{H} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathcal{P} \end{bmatrix} \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathfrak{L} & \mathcal{L} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1^H \\ \mathcal{Q}^H \end{bmatrix}, \tag{4}$$

where $\mathbf{H}_1$ and $\mathbf{Q}_1^H$ are $n_1 \times N_T$ submatrices, and $\mathbf{L}_1$ and $\mathbf{P}_1$ are $n_1 \times n_1$ square matrices. Expanding (4), we have the following two equations

$$\mathbf{H}_1 = \mathbf{P}_1\mathbf{L}_1\mathbf{Q}_1^H, \tag{5}$$

$$\mathcal{H} = \mathcal{P}\mathfrak{L}\mathbf{Q}_1^H + \mathcal{P}\mathcal{L}\mathcal{Q}^H. \tag{6}$$

From equation (5), we see that by using the Generalized Triangular Decomposition (GTD) [2], almost any desired diagonal elements in $\mathbf{L}_1$ can be obtained. In particular, the diagonal elements of this submatrix can be made equal by using the GMD, a special case of the GTD. Now, since $\mathbf{Q}$ is unitary, the submatrices $\mathbf{Q}_1$ and $\mathcal{Q}$ are orthonormal to each other. Thus, from equation (6), the projection matrix $\mathbf{I} - \mathbf{Q}_1\mathbf{Q}_1^H$ is used to get

$$\mathcal{H}(\mathbf{I} - \mathbf{Q}_1\mathbf{Q}_1^H) = \mathcal{P}\mathcal{L}\mathcal{Q}^H. \tag{7}$$

Here, the right side of (7) has the same form as equation (3), so we can proceed recursively. Finally, to solve for $\mathfrak{L}$, equation (6) is multiplied by $\mathcal{P}^H$ and $\mathbf{Q}_1$, giving

$$\mathfrak{L} = \mathcal{P}^H\mathcal{H}\mathbf{Q}_1. \tag{8}$$

We shall refer to the decomposition that achieves equal diagonal elements in each block of $\mathbf{L}$ as the block-diagonal geometric mean decomposition (BD-GMD).

C. *Diagonal Elements*

Consider a BD-GMD decomposition $\mathbf{H} = \mathbf{PLQ}^H$. Let the diagonal elements of the i-th block of $\mathbf{L}$ be r_i. To calculate each r_i, equations (4) and (5) are generalized to get

$$\begin{bmatrix} \widehat{\mathbf{H}}_i \\ \mathcal{H} \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{P}}_i & \mathbf{0} \\ \mathbf{0} & \mathcal{P} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{L}}_i & \mathbf{0} \\ \mathfrak{L} & \mathcal{L} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{Q}}_i^H \\ \mathcal{Q}^H \end{bmatrix}, \tag{9}$$

$$\widehat{\mathbf{H}}_i = \widehat{\mathbf{P}}_i\widehat{\mathbf{L}}_i\widehat{\mathbf{Q}}_i^H, \tag{10}$$

where the submatrices $\widehat{\mathbf{H}}_i$, $\widehat{\mathbf{P}}_i$, $\widehat{\mathbf{L}}_i$ and $\widehat{\mathbf{Q}}_i^H$ each have $\sum_{j=1}^{i} n_j$ rows. Because $\widehat{\mathbf{P}}_i$ and $\widehat{\mathbf{Q}}_i^H$ are unitary, (10) shows that the singular values of $\widehat{\mathbf{H}}_i$ and $\widehat{\mathbf{L}}_i$ must be the same. Thus,

$$\det(\widehat{\mathbf{H}}_i^H\widehat{\mathbf{H}}_i) = \det(\widehat{\mathbf{L}}_i^H\widehat{\mathbf{L}}_i) = \prod_{j=1}^{i} r_j^{2n_j}. \tag{11}$$

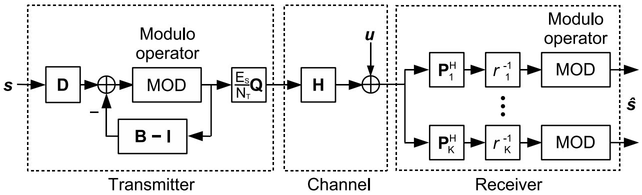

Figure 1: Block Diagram of the BD-GMD-DPC scheme implementing THP and user ordering.

Therefore, we have

$$r_i = \sqrt[2n_i]{\frac{\det(\widehat{\mathbf{H}}_i^H \widehat{\mathbf{H}}_i)}{\det(\widehat{\mathbf{H}}_{i-1}^H \widehat{\mathbf{H}}_{i-1})}}\,. \tag{12}$$

D. Ordering the Users

The diagonal elements of $\mathbf{L}$ are usually in decreasing order because $\mathbf{LQ}^H$ is the QR-decomposition of $\mathbf{P}^H\mathbf{H}$. The first element can often be many times that of the last one, so the first user enjoys much better performance than the last user. Equation (12) tells us that the diagonal elements r_i depend on the ordering of the rows of $\mathbf{H}$, or in other words, the ordering of the users. Our hope is to improve the fairness of the scheme by ordering the users to increase the size of the last few diagonal elements.

A method inspired by that used in the BLAST [6] system is used. First, note that $\det(\widehat{\mathbf{H}}_{i-1}^H \widehat{\mathbf{H}}_{i-1})$ does not change with the order of the rows of $\widehat{\mathbf{H}}_{i-1}$. Therefore, from (12), the value of r_K depends only on the choice of $\mathbf{H}_K$ and not the order of the first $K-1$ users. Thus, to maximize r_K, $\mathbf{H}_K$ is chosen to minimize $\det(\widehat{\mathbf{H}}_{K-1}^H \widehat{\mathbf{H}}_{K-1})$. Similarly, $\mathbf{H}_{K-1}$ is chosen to minimize $\det(\widehat{\mathbf{H}}_{K-2}^H \widehat{\mathbf{H}}_{K-2})$ and so on. Let the decomposition that optimizes the diagonal elements in such a way be called the *ordered* BD-GMD.

Let $\{\pi_1, \pi_2, \ldots, \pi_K\}$ be the optimal ordering of the users where what was previously the π_1-th user is now the first user, and so on. Since the ordering of the users result in the ordering of the rows of $\mathbf{H}$, this ordering may also be represented by a permutation matrix $\mathbf{D}$ such that $\mathbf{DH} = \mathbf{PLQ}^H$. Here, the i-th block $\mathbf{P}_i$ of $\mathbf{P}$ has dimensions $n_{\pi_i} \times n_{\pi_i}$.

IV. BD-GMD-DPC

In this section, the transceiver design method using BD-GMD, called BD-GMD-DPC, is presented.

A. Transceiver Design

Suppose $\mathbf{s} = [\mathbf{s}_1^T, \cdots, \mathbf{s}_K^T]^T$ is the unordered vector of information symbols to be sent, where $\mathbf{s}_i$ is the information symbol vector of user i. First, let $\mathbf{DH} = \mathbf{PLQ}^H$ be the ordered BD-GMD of $\mathbf{H}$. Write $\mathbf{L} = \mathbf{\Lambda B}$ with $\mathbf{\Lambda} = \mathrm{diag}(\mathbf{L})$, and $\mathbf{B}$ a lower triangular matrix with unit diagonal. Multiplying (1) by $\mathbf{D}$ gives

$$\widetilde{\mathbf{y}} = \mathbf{PLQ}^H\mathbf{x} + \widetilde{\mathbf{u}}, \tag{13}$$

where $\widetilde{\mathbf{y}}$ is the reordered received signal vector. Let $\widetilde{\mathbf{s}} = \mathbf{Ds}$ be the reordered information symbol vector. Using $\mathbf{x} = \mathbf{Q}\widetilde{\mathbf{s}}$ and $\widetilde{\mathbf{z}} = \mathbf{P}^H\widetilde{\mathbf{y}}$ for the transmit and receive equalization respectively, transforms the channel to

$$\widetilde{\mathbf{z}} = \mathbf{L}\widetilde{\mathbf{s}} + \widetilde{\mathbf{u}}'. \tag{14}$$

Now DPC is performed at the transmitter to pre-subtract the interference represented by $\mathbf{L}$. As a result, user π_i enjoys n_{π_i} independent and equivalent subchannels of the form

$$z = r_i s + u, \tag{15}$$

where r_i is the diagonal element of the i-th block of $\mathbf{L}$.

Suppose the total transmit power E_s is distributed equally among the M transmit antennas. Then, the achievable sum-rate for the scheme is given by

$$C = \sum_{i=1}^{K} n_{\pi_i} \log_2\!\left(1 + \frac{E_s}{N_0 N_T} r_i^2\right). \tag{16}$$

Fig. 1 shows the block diagram of a scheme that uses the ordered BD-GMD and THP as a suboptimal implementation of DPC. Here, $\mathbf{P}_i^H$ and $\mathbf{\Lambda}_i^H$ are the sub-blocks of $\mathbf{P}$ and $\mathbf{\Lambda}$.

To improve the performance of the scheme, different codebooks can be applied to each user. The base station only needs to inform each user which codebook to use before data transmission, depending on the rank of the user in the optimal ordering of all the users. The user then uses the same codebook for all his subchannels since the subchannels are equivalent. Compared to a ZF-THP scheme which uses multiple codebooks, one for each subchannel, the complexity of each mobile receiver is greatly reduced.

V. Equal Rate BD-GMD-DPC

While BD-GMD-DPC achieves equal rate for the sub-channels of each user, the achievable rates for different users are different. In this section, relying on transmit power control, ER-BD-GMD-DPC is proposed to achieve equal rate for all subchannels across all the users.

A. Mathematical Formulation

ER-ZF-THP [3] is the result of considering the following optimization problem:

$$\begin{array}{ll} \texttt{maximize} & \alpha \\ \texttt{subject to} & \mathbf{H}\check{\mathbf{F}} = \alpha\mathbf{J} \\ & \mathbf{J} \in \mathbb{L} \\ & \mathrm{Tr}(\check{\mathbf{F}}\check{\mathbf{F}}^H) \le E_s \end{array} \tag{17}$$

where $\mathbb{L}$ is the set of lower triangular matrices with unit diagonal. Here, $\check{\mathbf{F}}$ represents the precoding matrix and $\mathbf{J}$ the interference matrix with which THP is performed.

The power allocation for our broadcast situation can be formulated as the following optimization problem:

$$\begin{aligned} \texttt{maximize} \quad & \alpha \\ \texttt{subject to} \quad & \mathbf{AH}\widetilde{\mathbf{F}} = \alpha\mathbf{J} \\ & \mathrm{Tr}(\widetilde{\mathbf{F}}\widetilde{\mathbf{F}}^H) \le E_s \\ & \|\mathbf{A}(i,:)\| = 1 \quad \text{for } 1 \le i \le N_R \\ & \mathbf{J} \in \mathbb{L},\ \mathbf{A} \in \mathbb{B}, \end{aligned} \tag{18}$$

where $\mathbb{B}$ is the set of block-diagonal matrices of the form in (2). Here, $\widetilde{\mathbf{F}}$ represents the precoding matrix, and $\mathbf{A}$ the receive equalization matrix, which may not be unitary. We assume $\|\mathbf{A}(i,:)\| = 1$ for each row $\mathbf{A}(i,:)$ of $\mathbf{A}$ so that the channel noise is not amplified by $\mathbf{A}$. Write $\widetilde{\mathbf{F}} = \alpha\mathbf{F}$. The condition $\mathrm{Tr}(\widetilde{\mathbf{F}}\widetilde{\mathbf{F}}^H) \le E_s$ can then be expressed as

$$\alpha^2 \le \frac{E_s}{\mathrm{Tr}(\mathbf{FF}^H)}, \tag{19}$$

so maximizing α is the same as minimizing $\mathrm{Tr}(\mathbf{FF}^H)$. Thus, (18) is equivalent to the following optimization:

$$\begin{aligned} \texttt{minimize} \quad & \mathrm{Tr}(\mathbf{FF}^H) \\ \texttt{subject to} \quad & \mathbf{AHF} = \mathbf{J} \\ & \|\mathbf{A}(i,:)\| = 1 \quad \text{for } 1 \le i \le N_R \\ & \mathbf{J} \in \mathbb{L}, \mathbf{A} \in \mathbb{B}. \end{aligned} \tag{20}$$

Solving the Lagrangian of this problem gives

$$\mathbf{F} = \mathbf{Q}\mathbf{\Lambda}^{-1}, \quad \mathbf{A} = \mathbf{P}^H, \quad \mathbf{J} = \mathbf{L}\mathbf{\Lambda}^{-1}, \tag{21}$$

where $\mathbf{H} = \mathbf{PLQ}^H$ is the BD-GMD of $\mathbf{H}$, and $\mathbf{\Lambda} = \mathrm{diag}(\mathbf{L})$. We call the construction given in (21) the *equal-rate* BD-GMD. In the case where $n_1 = n_2 = \ldots = n_K = 1$, we have ER-ZF-THP.

B. Transceiver Design

The precoding matrix is $\widetilde{\mathbf{F}} = \alpha\mathbf{F} = \alpha\mathbf{Q}\mathbf{\Lambda}^{-1}$ where

$$\alpha^2 = \frac{E_s}{\mathrm{Tr}(\mathbf{FF}^H)} = \frac{E_s}{\mathrm{Tr}(\mathbf{\Lambda}^{-2})} \tag{22}$$

while the receive equalization matrix is $\mathbf{A} = \mathbf{P}^H$. This transforms the channel to $\mathbf{z} = \alpha\mathbf{Js} + \mathbf{u}'$. DPC is then done at the transmitter to cancel the interference. As a result, every user enjoys independent and equivalent subchannels of the form

$$z = \alpha s + u. \tag{23}$$

The achievable sum-rate for the scheme is given by

$$C = N_R \log_2(1 + \frac{\alpha^2}{N_0}). \tag{24}$$

Hence, given a fixed order of users for dirty paper coding at the base station, the above equal-rate BD-GMD scheme optimizes the zero-forcing linear beamforming vectors and power allocation required to achieve maximum throughput and equal rates for every subchannel of every user.

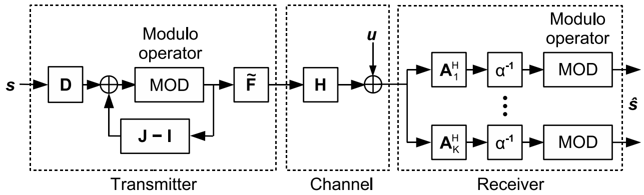

Figure 2: ER-BD-GMD-DPC implementing user ordering and THP.

In (22), the diagonal values $r_1, \ldots, r_{N_R}$ of $\mathbf{\Lambda}$ must satisfy $r_1 r_2 \ldots r_{N_R} = \det(\mathbf{H})$. Given this condition, the channel gain α is maximized when $r_1 = \ldots = r_{N_R}$. In general, α increases when we decrease the spread among $r_1, \ldots, r_{N_R}$. Thus, we can expect some performance gain by performing the *ordered* BD-GMD to reduce this spread. Fig. 2 shows the block diagram of a scheme that performs the equal-rate ordered BD-GMD and THP to implement DPC.

VI. Simulation Results

In this section, computer simulation results are presented to compare the performance of BD-GMD-DPC with ZF-THP, as well as their equal-rate versions. We consider the case of $12 \times \{4, 4, 4\}$ broadcast channel. We assume that the elements of the channel matrix $\mathbf{H}$ are independent complex Gaussian random variables with zero mean and unit variance.

In computing the BER performance, THP is used as a suboptimal implementation of DPC, and power is scaled down by a factor of $(M-1)/M$ (for M-QAM modulation) to account for the slightly higher average power required by THP [5].

Fig. 3 shows the performance comparison for different schemes with and without ordering. Here "uo" and "o" denote "unordered" and "ordered", respectively and 16-QAM is used for all the subchannels. It is seen that for the BD-GMD-THP and the ER-BD-GMD-THP, there is a 1 dB improvement at BER of 10^{-4} over their unordered counterparts. For ER-ZF-THP, ordering the users provides a very large gain of 6 dB at BER of 10^{-3}. The gain in BER performance is because ordering improves the channel gain of the worst subchannel, and the average BER performance is due primarily to the performance of the worst subchannel.

Fig. 4 shows the BER improvement coming from equalization at the receiver. All of the schemes perform user ordering. The notation "16" means that 16-QAM is used for every user, while "mc" means that multiple constellations are used, with the user with the largest channel gain being assigned 64-QAM, the next user 16-QAM, and the last user 4-QAM.

The solid line denoting the BD-GMD-THP(o,16) scheme is lower than the dashed line denoting the ZF-THP(o,16), showing a BER performance gain of more than 6 dB at BER of 10^{-3}. It can be seen that the BD-GMD schemes have a higher slope than the ZF-THP schemes. This is due to the higher diversity gain achieved by allowing equalization at the receivers.

The ER-BD-GMD-THP(o,16) scheme has a BER performance gain of 2.6 dB over the BD-GMD-THP(o,16) at BER of 10^{-4}. This is because the performance of the worst sub-

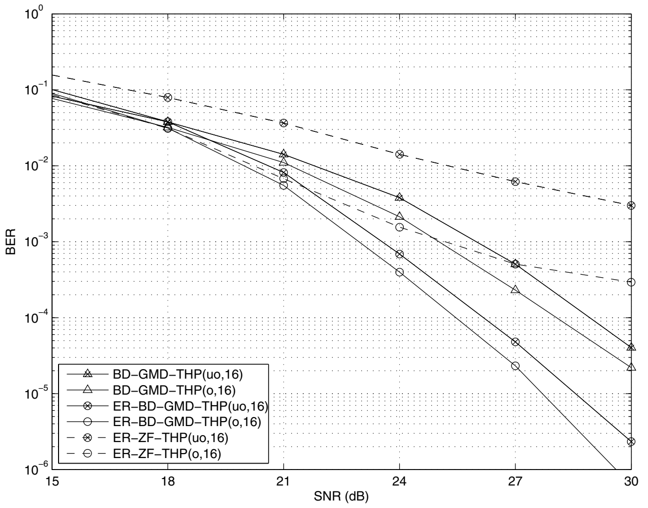

Figure 3: BER performance of ordered and unordered schemes.

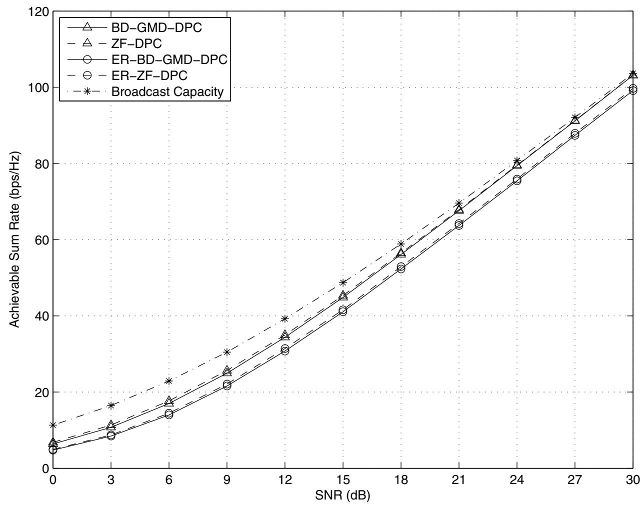

Figure 5: Achievable sum rates of proposed schemes.

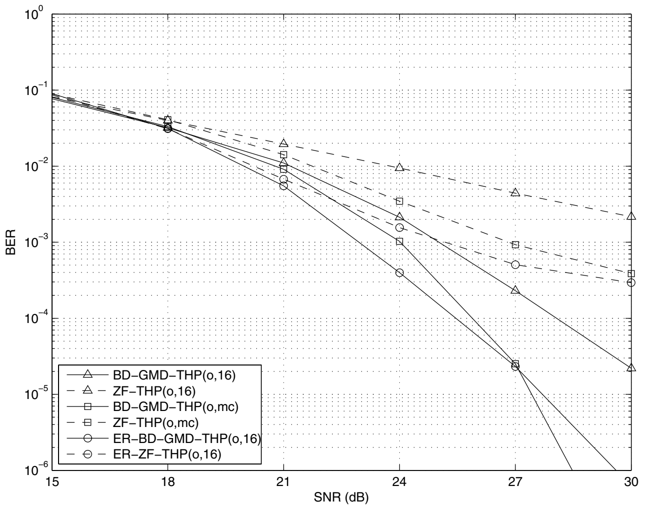

Figure 4: Effect of receiver equalization and multiple constellations on BER performance.

channel is mitigated by the equal-rate property. The BD-GMD-THP(o,mc) has a gain of 2.2 dB over BD-GMD-THP(o,16) because it allocates only 4-QAM for the weakest user.

Fig. 5 shows the achievable sum rates for ER and non-ER schemes, where all schemes are ordered and assume perfect DPC with Gaussian input. There is a slight loss in achievable rate when we demand fair treatment of the users via the ER schemes. This is because the ER schemes use channel inversion for power control.

VII. Conclusion

In this paper, we have presented the block-diagonal GMD for the multiuser MIMO downlink. Two schemes based on this BD-GMD have been proposed, using DPC at the transmitter – BD-GMD-DPC and ER-BD-GMD-DPC. The first scheme allows equal-rate coding for the subchannels of each user, while the second scheme allows equal-rate coding for every subchannel of every user.

Improvements in performance of the schemes was achieved by user ordering, use of multiple constellations and solving for optimal power control. Simulations have showed that both the two schemes have better BER performance than ZF-THP and ER-ZF-THP, respectively.

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